A3_2 There's 24 Hours in a Day

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Abstract

One day on Earth is considered to have 24 hours. However, the time period of rotation of Earth on its axis is 23 hours 56 minutes and 4 seconds. This rotation period is gradually slowing down due to the recession of the Moon from Earth. In this paper we determine the year in which the rotation period of Earth would be exactly 24 hours. We find that when the average lunar orbital distance is 386,317km, then Earth would have a day comprising of 24 hours. This would take place 50.4 million years from now.

Introduction

On Earth we use a 24 hour day for convenience, in reality the Earth actually takes 23 hours 56 minutes and 4 seconds to do 1 complete rotation on its own axis [1].

The rotation of the Earth occurs due to the conservation of angular momentum from when the solar system formed around 4.6 billion years ago. This rotation is affected by other bodies in the solar system but these are negligible in comparison to the effect from the Moon. When the Moon was originally formed around 4 billion years ago it stabilized the Earth's orbit and gave it the 23 degree tilt that gives us our seasons. The Moon was also much closer to the Earth when it formed and it has slowly been receding ever since its formation, to its current position of 384,400 km from Earth and as a result the length of day on Earth has slowly been increasing.

Theory

We used the conservation of angular momentum to determine the new average orbital distance to the Moon for a 24 hour day on Earth. First we determine the current orbital velocity of the Moon. To find this we used Eq (1)

$$v_m = \sqrt{\frac{GM_E}{R_m}} \qquad (1)$$

where v_m is the orbital velocity of the Moon ($1020 \, \mathrm{ms^{-1}}$), G is the gravitational constant, M_E is the mass of the Earth and R_m is the average distance between the Earth and Moon.

We also calculated the surface velocity of the Earth. To find this we used Eq (2)

$$v_E = \frac{2\pi R_E}{t} \qquad (2)$$

where v_E is the surface velocity of the Earth, R_E is the radius of the Earth and t is the time for the Earth to make one rotation on its axis, this was evaluated for a current day length ($465 \, \mathrm{ms^{-1}}$) and a 24 hour day ($463 \, \mathrm{ms^{-1}}$).

Using the calculated velocities above we were able to calculate the angular momentum of the Earth and Moon using Eq (3)

$$L = MvR \qquad (3)$$

where L is the angular momentum of the object, M is the object's mass, v is the velocity of the object and R is the distance of the object from the centre of mass of the system. This was evaluated for the Earth at its current rate of rotation (1.78×10^{34} kg m^2 s^{-1}) and when the Earth's orbit will be 24 hours (1.77×10^{34} kg m^2 s^{-1}). The orbital angular momentum for the Moon at its current position was also calculated (2.88×10^{34} kg m^2 s^{-1}).

According to the law of conservation of angular momentum, the total angular momentum of the system would be the same before and after the recession of the moon. This is shown mathematically in Eq (4)

$$L_{E_1} + L_{m_1} = L_{E_2} + L_{m_2} \qquad (4)$$

where L_{E_1} is the Earth's initial rotational angular momentum, L_{m_1} is the Moon's initial orbital angular momentum, L_{E_2} is the Earth's final rotational angular momentum and L_{m_2} is the Moon's final orbital angular momentum. This was used to find the orbital angular momentum of the Moon after its recession (2.89×10^{34} kg m^2 s^{-1}). Substituting Eq (1) into Eq (3) produces Eq (5).

$$L_{m_2} = M_m \sqrt{GM_E R} \qquad (5)$$

Making the distance the subject of the equation, we attain Eq (6)

$$R = \frac{L_{m_2}^2}{GM_E M_m^2} \qquad (6)$$

Substituting in values we determine the distance at which the Moon would have to be from Earth to give Earth a 24 hour day. This distance was calculated to be 386,317 km. From this distance we could determine how long it would take for the Moon to recede to this distance given that it is receding at the rate of 3.8 cm per year [2] this value came out as 50.4 million years.

Discussion

In this paper, we have calculated the distance that the Moon will have to recede to in order to slow down the rotational speed of the Earth to make a day last exactly 24 hours, we have also calculated how long this will take at the Moon's current rate of recession. In order to calculate this we have made a number of assumptions.

Firstly we assumed that the recession rate of the Moon is constant and effects all parts of the Moons orbit equally. We also assumed that other bodies in the solar system have a negligible effect on the rotation of the Earth. In addition, we assumed that the centre of mass of the system was constant and is at the centre of Earth. This assumption was made since the distance from the true centre of mass to the centre of mass of the Earth is relatively small compared to the Earth-Moon distance.

We also neglected the Moon's rotational angular momentum as it is negligible compared to the Earth's and Moon's orbital angular momenta.

Conclusion

In this paper, we have considered the recession of the Moon and have calculated the recession distance required in order to slow down the rotation of the Earth so that a day lasts 24 hours. We found that the Moon's average orbital distance would have to increase by 1917 km and that at current recession rates, this will take 50.4 million years. This value is different to the estimates of the rate of day lengthening shown in [2]. This difference occurs mainly due to the assumption that the centre of mass of the system is at the centre of the Earth, whereas in reality it is not and as a result the actual value would be smaller than that calculated in this paper.

Further work could be done to determine how much of an effect using the exact centre of mass of the system would effect the time of the Moon's recession to the distance where Earth would have a 24 hour day.

References

[1] `goo.gl/mmjzEj` accessed on 14/10/2016

[2] `goo.gl/ReQSlw` accessed on 14/10/2016