

## A6\_4 Lunar View of La Bombonera

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### Abstract

We use principles from diffraction to estimate the required size for a reflecting telescope to watch an association football match at La Bombonera from the Moon. We determine that a mirror of diameter 1.13 ( $\pm 0.08$ ) km with focal length 17.00 ( $\pm 1.15$ ) km at focal ratio  $f/15$  will be needed.

### Introduction

It used to be suggested that a telescope could be trained on the Moon to observe American flags left by Apollo astronauts. However, this is far from possible due to the resolution of optical telescopes being limited by issues such as diffraction and atmospheric seeing effects.

We spin this idea and apply it to a sporting event. How large would a reflecting telescope have to be to watch a football match at La Bombonera, the home ground of Argentine club side CA Boca Juniors?

### Theory

Newtonian telescopes use a primary parabolic mirror that focusses light from a distant source to a single focal point. A small plane mirror set at a  $45^\circ$  angle to the primary at the focal point further reflects this light into an eyepiece.

We assume that the technology to build a telescope on the Moon exists; that it is daytime on Earth; that the atmospheres of both bodies induce no seeing effects; and, that the smallest object we would have to resolve is a size 5 football of circumference 70 ( $\pm 1$ ) cm [1].

As per Figure 1, we can estimate the angle

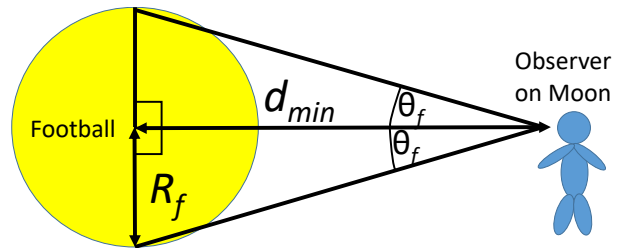


Figure 1: This demonstrates the trigonometry for estimating the angular width of the football. Not to scale.

subtended by half the football,  $\theta_f$ , using Equation 1:

$$\tan \theta_f = \frac{R_f}{d_{min}} \quad (1)$$

where  $R_f = 11.15$  ( $\pm 0.16$ ) cm is the football radius and  $d_{min}$  is the closest distance between the surfaces of Earth and Moon. Note that  $\tan \theta_f \approx \theta_f$  due to the small-angle approximation. Subsequently,  $2\theta_f$  will be the angle subtended by the whole football.

$d_{min}$  is found using Equation 2:

$$d_{min} = d_{EM} - (R_E + R_M) \quad (2)$$

where  $d_{EM} = 382,000$  ( $\pm 25,000$ ) km is the average separation between the centres of both Earth and Moon [2];  $R_E = 6,367.45$  ( $\pm 10.65$ ) km and

$R_M = 1737.05 (\pm 1.05)$  km are the average Earth and lunar radii, respectively [2]. These averages account for the lunar elliptical orbit and both bodies being oblate spheroids [3].

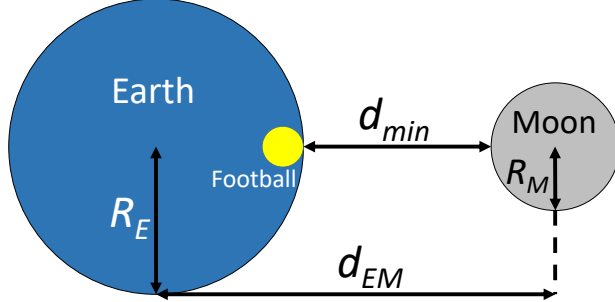


Figure 2: Shows the distances involved in determining  $d_{\min}$ . Not to scale.

The diameter,  $D$ , of the aperture opening (hence primary mirror) of the telescope can be estimated by using Equation 3 [4]:

$$\sin \theta = \frac{1.22\bar{\lambda}}{D} \quad (3)$$

where  $\bar{\lambda} = 550$  nm is the average wavelength of visible light [5] and  $\theta$  is the angle subtended by the first diffraction minimum of a circular aperture (in radians) [4]. Note that  $\sin \theta \approx \theta$  due to the small-angle approximation.

The focal ratio,  $F$ , of a telescope is the ratio between the focal length of the primary mirror and its effective aperture [6]. Often denoted as  $f$ /number, it can be calculated using Equation 4:

$$F = \frac{f}{D} \quad (4)$$

where  $f$  is the telescope focal length and  $D$  is the diameter of the mirror.

## Results & Discussion

If  $d_{\min} = 373,896 (\pm 25,000)$  km, then  $\theta_f = 2.98 \times 10^{-10} (\pm 2.04 \times 10^{-11})$  rad. The diameter,  $D$ , of the primary mirror is found by substituting for  $\theta = 2\theta_f$  in Equation 3 and rearranging to give  $D = 1.13 (\pm 0.08)$  km.

The uncertainty in the diameter estimate does not account for any uncertainty in  $\bar{\lambda}$ . This is because reflecting telescopes do not suffer from

chromatic aberration since reflected visible light does not disperse due to its wavelength [7].

The focal length of the telescope is estimated using the linear relationship of Equation 4. A small focal ratio gives a wide field of view (FOV) [8]. Conversely, large  $f$ /numbers provide more magnification with the same eyepiece, but a smaller FOV, making it easier to get high magnification for planets [9].

$f/4$  to  $f/5$  ratios are best for low power wide field observing, while  $f/11$  to  $f/15$  ratios are suited to high power lunar and planetary observing [8].  $f/6$  to  $f/10$  ratios work well with both [8].

Based on the above, a large  $f$ /number would suit this situation. Thus, if we choose  $f/15$  ( $F = 15$ ), the focal length is  $f = 17.00 (\pm 1.15)$  km.

## Conclusion

It would be incredibly impractical to build a telescope of this size anywhere. However, impracticalities aside this problem can spawn further work. Future considerations could include: the auto-guiding requirements to track a game as the Earth and Moon rotate on their respective axes; determining the size of eyepieces and their respective magnifications; or, calculating the same specifications for a refracting telescope.

## References

- [1] <https://goo.gl/4hwICS> accessed 16/10/16.
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- [4] Tipler, P. A. and Mosca, G.; Physics for Scientists and Engineers; New York: W. H. Freeman, 2008; 6th Edition, page 1160.
- [5] Moore, P.; Astronomy encyclopedia; London: Philip's, 2002; page 432.
- [6] Moore, P.; Astronomy encyclopedia; London: Philip's, 2002; page 146.
- [7] <https://goo.gl/o7T5s3> accessed 26/10/16.
- [8] <https://goo.gl/vPrTmV> accessed 25/10/16.
- [9] <https://goo.gl/SKae9u> accessed 25/10/16.