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A2_1 How to Train Your Kraken

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Abstract

We investigate wave propagation through water and its frequency dependance to determine the maximum distance Davy Jones' Kraken can be from the Flying Dutchman and still hear his summoning call. We found the range to be 3.2 ± 0.7 km, which is much less than expected as the Flying Dutchman is unlikely to be able to produce sounds loud enough for a greater range.

Introduction

In Pirates Of The Caribbean: Dead Man's Chest, Davy Jones summons a Kraken by using a large mechanical device to send out a low frequency sound wave. In this paper we will develop a model to investigate the frequency dependence of the amplitude of the sound wave as it travels through the water. We will use this to constrain how far away the Kraken can be and still hear the signal.

Theory

As the sound wave travels through the water it loses intensity via two processes. The first of these is sound attenuation which occurs due to thermal absorption as the wave propagates through a fluid, this is an exponential decay. The second comes from conservation of energy. This means the intensity also follows an inverse square law. The intensity can then be calculated using

$$\frac{I_f}{I_i} \approx \frac{e^{-2\alpha r}}{r^2}, \tag{1}$$

where I_i and I_f are the initial and final intensities [1], r is the distance from the source and α is the attenuation coefficient [2] given by

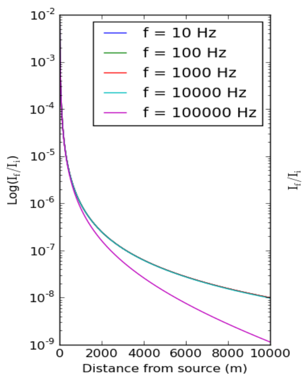
$$\alpha = \frac{2\eta\omega^2}{3\rho v^3}. \tag{2}$$

Here η is the coefficient of viscosity, ω is the angular frequency, ρ is the density of water and v is the wave speed (speed of sound in water). Eq (2) assumes that water is an incompressible fluid. Eq (1) and Eq (2) allows us to investigate the dependance of relative intensity on frequency and distance from the source. This is shown in Fig (1).

It is clear from Fig (1) that the exponential term dominates at large distances, meaning that attenuation cannot be neglected for long range communication. Fig (2) depicts the frequency dependance of the relative intensity at a fixed distance which shows that the drop in intensity due to increasing frequency is only of order 10^{-10} at 100km. Both figures suggest that whilst the intensity does depend on frequency, this effect is only important for frequencies of order 10^5 Hz.

Results

The assumptions for this scenario are that the signal source is as loud as thunder (130dB) and



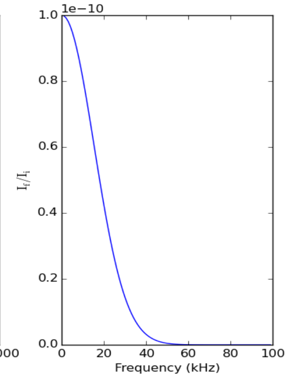


Figure 1: The change in relative intensity of the sound wave against the distance from the source at varying frequencies from 10Hz-100kHz. (Left)

Figure 2: Change in relative intensity of the sound wave for varying frequencies at a distance 100km. (Right)

must be as loud as a voice (60dB) by the time it reaches the Kraken to be heard above background noise. Next we must relate the intensity to decibels in order to determine the range. This is given by Eq (3)

$$dB = 10\log_{10}(I/I_{ref}) \tag{3}$$

where I is the wave intensity and I_{ref} is the reference intensity [3], which is 1×10^{-18} Wm^{-2} for water. We also assume that the Kraken has a similar hearing range to that of a whale (10 Hz - 100 kHz) [4]. Given these constraints on initial and final intensities, Eq (3) allows us to determine that the wave intensity can drop by a factor of 10^7 before the sound wave is inaudible to the Kraken. I.e $I_f/I_i = 1.0 \times 10^{-7}$. Using this, Fig (1) gives a graphical solution as a range of 3.2 km for the lowest audible frequency (10 Hz) which will suffer the lowest losses over a given range.

Error Analysis

In this model we assume a fixed ocean temperature of 288K, and used the corresponding values of η, ρ and v at this temperature. However, there will be an error associated with each of these values due to the range of possible temperatures of the ocean. We account for this by appending the following errors, which come from considering the temperature dependance of the system; $\eta = 1.29 \times 10^{-4} \pm 4.95 \times 10^{-4}$ Nsm^{-2}, $v = 1450 \pm 50.0$ ms^{-1} and $\rho = 1030 \pm 3.30$ kgm^{-3}. For a frequency of 10 Hz, standard error propagation gives the uncertainty in I_f/I_i as $1.0 \times 10^{-7} \pm 3.96 \times 10^{-8}$. Using the uncertainty in I_f/I_i in conjunction with the graphical solution provided by Fig (1) gives the range and its uncertainty as 3.20 ± 0.70 km.

Conclusion

We have shown that the Kraken will hear its summoning call as long as it is not further than 3.2 ± 0.7 km away. In contradiction to this value, whales can be known to communicate over ranges up to 1600 km [5]. This is because some whales are able to produce sounds as loud as 200 dB. This means the main restriction here is the ability to generate a loud enough sound in the first place. It would be unreasonable to assume that the mechanism used to summon the Kraken is capable of creating sound waves much louder than thunder due to its portrayal in the film.

References

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