Journal of Physics Special Topics
An undergraduate physics journal

P2 3 Quantifying Alexander's Dark Band in the Dual Crossed Rainbow

N. Khatun, K.B. Meadows, R. Rashid, I. Davies

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

October 26, 2026

Abstract

When two rainbows cross, the dark regions between the primary and secondary bows, termed the *Alexander's dark band*, overlap. This paper uses a simplified additive luminance model to assess whether this intersection is darker or brighter than one dark band. Assuming a uniform sky background of 100 cd m^{-2} and an extinction contribution of 10 cd m^{-2} per dark band, the intersection luminance is 80 cd m^{-2}, compared with 90 cd m^{-2} for a single band. As rainbow systems are incoherent, their contributions add rather destructively interfere. Under these assumptions, the intersection is darker than a single dark band, although enhanced contrast with neighbouring bright bows could make it appear darker. Realistic predictions would require measured luminances or an angular scattering model accounting for illumination, droplet properties and viewing geometry.

Introduction

Always a marvel from the ground, the rainbow remains a striking example of how sunlight interacts with water droplets, producing vivid arcs of colour through refraction, internal reflection, and dispersion. When light strikes a droplet's surface, the displacement of the incident ray from air to the drop's central axis, changes its direction. This is defined by Snell's law of reflection and refraction at a spherical surface [1].

On the droplet's surface, the incident ray is partially reflected, and the scattered rays are considered Class 1 [2]. The remaining light is partially transmitted at the next surface, and partially reflected, identified as Class 2. Class 3 rays are internally reflected and form the primary rainbow by escaping the droplet [2]. Class 4 rays undergo two internal reflections and form the secondary bow. The Class 3 scattering angle passes through a minimum of 138° [3], and Class 4 scattering angles have a maximum of 130° [2]. The angular region, approximately 8° between Class 3 and 4, is named Alexander's dark band [2]. This paper attempts to quantify how dark Alexander's dark band would be when two rainbow's cross each others path in the sky.

Assuming the Sky's Luminance

The rainbow inspiring this paper was spotted on 25th September 2026 at 18:55 BST in Manchester. At Manchester's latitude, the Sun was expected at the horizon, with sunset estimated at 19:00 BST [4]. That makes civil twilight reasonable; rather than a dark, night sky, the observation would have been during a dim, but still illuminated sky [5].

As a modeling assumption, we can presume a rain shower ahead, a clear western horizon that allows direct sunlight, and a locally uniform,

dim sky background with luminance: $L_{\mathrm{bg}} = 100\ \mathrm{cd\,m^{-2}}$.

This is an arbitrary number for simplification. We can test the plausibility of it using the diffuse horizontal illuminance E_h, calculated by integrating the sky luminance over the upper hemisphere, projected onto a horizontal plane [6]. If we assume the sky luminance is uniform, the integral simplifies from:

$$E_h = \int_0^{2\pi} \int_0^{\pi/2} L(\theta, \phi) cos(\theta) sin(\theta) d\theta d\phi \quad (1)$$

to: $E_h = \pi \cdot L_0$.

Taking L_0 equal to L_{bg}, E_h is approximately 314 lux, as expected at the start of civil twilight [7]. This is a plausible illustrative illumination level near sunset; cloud cover and viewing direction could affect it.

Assuming the Rainbow's Luminance

The primary bow is the brightest optical feature of the rainbow. To be seen, its luminance must exceed the sky's. For this paper, assume the primary bow's luminance is: $L_{\mathrm{p}} = 150\ \mathrm{cd\,m^{-2}}$.

According to Schaaf [8], the secondary bow is 1/10th the intensity of the primary bow, making the luminance of the secondary bow: $L_{\mathrm{s}} = 15\ \mathrm{cd\,m^{-2}}$.

Therefore, the luminance of Alexander's dark band must be lower than that of the secondary bow, as it is described as being significantly darker than the sky outside the bows and the main arch [9]. So assume Alexander's dark band luminance, as an extinction contribution is: $L_{\mathrm{Ab}} = (-)10\ \mathrm{cd\,m^{-2}}$.

Again, these are arbitrary luminance values, for this simple model to work. Physical measurements of the primary bow, secondary bow, Alexander's dark band and the sky's background could not be taken.

The Luminance of the Intersection

Treating the given bow luminances as additional light above the sky's background, the dark band would still receive background light, so it would not be completely black. The total primary bow luminance is:

$$L_{\mathrm{Ptotal}} = L_{\mathrm{bg}} + L_{\mathrm{p}} = 250\ \mathrm{cd\,m^{-2}} \quad (2)$$

The total secondary bow luminance is:

$$L_{\mathrm{Stotal}} = L_{\mathrm{bg}} + L_{\mathrm{s}} = 115\ \mathrm{cd\,m^{-2}} \quad (3)$$

When adding a second rainbow, you might assume that the light would cancel out through destructive interference [1] but that requires waves with a stable phase relationship (coherent light). Rainbow light comes from many separate raindrops [2], so those phase relationships aren't maintained, and any interference averages out, leaving their intensities adding:

$$I_{total} \approx I_1 + I_2... \quad (4)$$

We can add the luminances without converting to intensities, assuming incoherent light [10].

The luminance of the intersection considers the weak contribution of a second Alexander's dark band, and the sky's background, becoming:

$$L_{\mathrm{intersection}} = L_{\mathrm{bg}} + 2(L_{\mathrm{Ab}}) = 80\ \mathrm{cd\,m^{-2}} \quad (5)$$

Including the sky's luminance, the overlapping dark band, would be 68% darker than the primary bow, 30.4% darker than the secondary bow, and 11.1% darker than a single Alexander's dark band.

Discussion

In this simplified model, the overlapping Alexander's dark band is darker than a single band: the rainbows' weak contributions add at the intersection, but extinction reduces luminance against the shared sky background. We assume equal contributions and counts the background once. However, observers may see the crossing appear brighter, which might just be visual contrast: as nearby primary and secondary bows brighten together, the intersection may seems brighter even as its own luminance falls. In reality, luminance depends on droplet illumination and size, cloud cover, and bow geometry; a realistic estimate would require measured luminances or an angular scattering model.

References

[1] Tipler, P.A. and Mosca, G. (2007). Physics for scientists and engineers. New York, Ny: W.H. Freeman.

[2] Nussenzveig, H.M. (1977). The theory of the rainbow. *Scientific American*, 236(4), pp.116–128. Available at: `https://www.jstor.org/stable/24953986` [Accessed 11 Oct. 2026].

[3] Adam, J.A. (2002). The mathematical physics of rainbows and glories. *Physics Reports*, 356(4), pp.229–365. doi:10.1016/S0370-1573(01)00076-X.

[4] Timeanddate.com (2026). *Sunrise and sunset times in Manchester, September 2026*. [online] Available at: `https://www.timeanddate.com/sun/uk/manchester?month=9&year=2026` [Accessed 11 Oct. 2026].

[5] US Department of Commerce, NOAA (n.d.). *Twilight types*. [online] National Weather Service. Available at: `https://www.weather.gov/lmk/twilight-types` [Accessed 11 Oct. 2026].

[6] Kocifaj, M., Posch, Th. and Solano Lamphar, H.A. (2014). On the relation between zenith sky brightness and horizontal illuminance. *Monthly Notices of the Royal Astronomical Society*, 446(3), pp.2895–2901. doi:10.1093/mnras/stu2265.

[7] Visualexpert.com (2024). *Illuminance as a visibility criterion: the twilight envelope (3.2 lux)*. [online] Available at: `https://www.visualexpert.com/Resources/twilightcriterion.html` [Accessed 11 Oct. 2026].

[8] Schaaf, F. (1983). *Wonders of the Sky: Observing Rainbows, Comets, Eclipses, the Stars, and Other Phenomena*. New York: Dover Publications.

[9] Steele, P. (2026). *The physics of the double rainbow: what is Alexander's band?* [online] BaldHiker. Available at: `https://www.baldhiker.com/physics-of-double-rainbow-alexanders-band/` [Accessed 11 Oct. 2026].

[10] Deng, Y. and Chu, D. (2017). Coherence properties of different light sources and their effect on the image sharpness and speckle of holographic displays. Scientific reports, [online] 7(1), p.5893. doi:10.1038/s41598-017-06215-x.