Journal of Physics Special Topics
An undergraduate physics journal

P1 2 Butter-side Down

S. Hyde, M. Alquraishi, N. Hazlewood, A. McCorkindale

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

October 27, 2026

Abstract

A common superstition in modern life is that freshly buttered toast has a tendency to land butter-side down. This paper suggests a kinematic model to predict the starting height required for this statement to always be true by calculating the torque produced due to an offset centre of mass, and furthermore, simulate the model to determine that a starting height of $42.48m$ and fall duration of $6.37s$ is required to guarantee it.

Introduction and theory

Contrary to popular belief, freshly buttered toast is not guaranteed to fall butter-side down. Other papers [1] have determined that the initial angular and horizontal velocity conditions typically dictate if the toast reaches that tragic fate, however, there are more factors to consider.

We will model the toast from a rest position and instead account for the offset centre of mass by treating the butter as an additional, infinitely thin, layer that shifts the centre of mass away from the geometric centre. As the toast falls, an atmospheric drag force will be applied at the centre of pressure which differs from the centre of mass to produce a small torque calculated as:

$$\tau = rF_d \cos(\theta_p) \tag{1}$$

Where r is the distance between the centres of mass and pressure and $F_d \cos(\theta_p)$ is the drag force component that acts perpendicular to r.

Angular acceleration α is then found as [2]:

$$\alpha = \frac{\tau}{I} \tag{2}$$

With the value of I as the moment of inertia of the toast. For this we modelled the toast, of length $a = 12cm$ and width $b = 10cm$, as if the butter is a hypothetical additional volume extending the thickness (originally $c = 1.5cm$) while matching the density of the bread. We can then calculate for an ideal rectangular prism [2]:

$$I = \frac{1}{12} m(a^2 + (c + c_{butter})^2) \tag{3}$$

Where m is the estimated mass of the toast ($40g$) and butter ($3g$) combined.

The drag force from equation (1) cannot be perfectly predicted due to the complexity of the model and therefore will repeatedly require recalculation using the equation [2]:

$$F_d = \frac{C_d \rho_a A v^2}{2} \tag{4}$$

Where C_d is the coefficient of drag of the system, $\rho_a = 1.225kgm^{-3}$ is the atmospheric density at $15°C$ and $101kPa$ [3], A is the 2D horizontal planar surface area of the system and v is the vertical velocity of the toast in freefall.

The value of C_d, similarly to F_d, is difficult to predict due to the fact that both values are inextricably linked and would typically require experimental data. The best approximation is to

use a known value $C_d \approx 1.15$ [4] for a smooth rectangular plate at terminal velocity with a maximum drag force of [2]:

$$F_d = mg \tag{5}$$

Rearranging equations (4) and (5) for velocity we get $v_T = 7.09ms^{-1}$ which can then be compared to the velocity during freefall to approximate C_d.

Using the flawed assumption that the toast's coefficient of drag scales only in relation to its 2D surface area and velocity we can determine the C_d of the system at a given time to be:

$$C_d \approx 1.15\frac{v^2 A}{v_T^2 A_T} \tag{6}$$

Assuming the toast only rotates on the axis that acts parallel to the width of the slice and the angle θ is 90°$\leq \theta <$180° from the horizontal, the area A can be calculated as:

$$A = b(a\cos(180-\theta) + c\cos(\theta - 90)) \tag{7}$$

This model expects that once the slice of toast reaches an angle of $\theta = 90°$ any additional rotational motion and forces, including an early collision with the ground, will eventually settle into a stable system with the butter-side facing down ($\theta = 0°$). Additionally, angles greater than $\theta = 180°$ would produce a symmetrically structured system and have been disregarded.

The value of r used in equation (1) should be determined by finding the magnitude of vectors:

$$r_{CoP}(x) = \frac{(a\cos(180-\theta) + c\cos(\theta - 90))}{2} \tag{8}$$

$$r_{CoP}(y) = r_{CoP}(x)\tan(180-\theta) \tag{9}$$

$$r_{CoM}(x) = r_{CoP}(x) - \frac{c_{butter}}{2}\tan(180-\theta) \tag{10}$$

$$\begin{aligned} r_{CoM}(y) = (&\frac{c_{butter}+c}{2\sin(180-\theta)} - c\cos(\theta-90) \\ &+ r_{CoM}(x))\tan(180-\theta) \end{aligned} \tag{11}$$

Using the Euler integration method, we take a significantly small time step value $\Delta t = 0.0001s$ to increment the system gradually, recalculating A, C_d and F_d each step using equations (5), (4) and (2) respectively. We can then progress the motion of the system by calculating the step displacement s and velocity v using equations [2]:

$$a = \frac{mg - F_d}{m} \tag{12}$$

$$s = u\Delta t + \frac{1}{2}a\Delta t^2 \tag{13}$$

$$v = u + a\Delta t \tag{14}$$

Where u is the previous steps velocity value and a is the modified acceleration accounting for drag force opposing gravitational acceleration.

Finally we can progress the rotational phase of the system in steps of Δt using the angular acceleration calculated in equation (2) to find [2]:

$$\omega = \omega_0 + \alpha\Delta t \tag{15}$$

$$\theta = \theta_0 - \omega\Delta t - \frac{1}{2}\alpha\Delta t^2 \tag{16}$$

With θ_0 and ω_0 values equal to the previous steps angle to the horizontal in radians and angular velocity respectively.

Discussion and Conclusion

By repeating these steps we can determine for a slice of toast starting at $\theta_0 = 179.999$°(if theta is 180° our model suggests the toast experiences no rotation due to evenly distributed pressure which would not be true practically) the starting height required to guarantee landing butter-side down is $s = 42.48m$ and the total time elapsed for the almost 90° rotation is $t = 6.37s$.

As θ approaches 90°, the centre of pressure will eventually no longer sit on the length side of the toast. This invalidates a few of the vector calculations, however, as it only occurs in the last few steps ($0.0403s$ left) of the simulation it does not add significant error and we can counteract the more egregious values of r by capping those values at $r \approx a/2$. Additionally, air pressure would not evenly distribute itself and motional influences from wind have not been accounted for, nor the additional axis of rotation the toast may have. This model is too flawed for real world application as many variables have been approximated rather than tested experimentally.

References

[1] Matthews, R.A., *Tumbling toast, Murphy's Law and the fundamental constants.* European Journal of Physics, 16(4), pp.172-176. (1995)

[2] P. A. Tipler, G. Mosca. *Physics for Scientists and Engineers*, 6th Edition (New York, 2008)

[3] `https://en.wikipedia.org/wiki/International_Standard_Atmosphere` [Accessed 10/10/26]

[4] `https://www.sciencedirect.com/topics/engineering/drag-coefficient` [Accessed 10/10/26]