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A2 5 Equatorial Auroras

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Abstract
Auroral emission is normally restricted to high latitudes, reflecting the boundary between closed and open geomagnetic field lines. Using a simple dipole model of Earth's magnetic field, we compute the total open magnetic flux in a polar cap as a function of its boundary latitude. We then apply a visibility criterion to determine how much open flux would be required for aurora to reach low latitudes or the equator. The model shows that equatorward auroras require 14.3 times more open flux than typical geomagnetic conditions.

Introduction

The polar caps are regions surrounding the geomagnetic poles where the magnetic field lines are open to the solar wind (connected to the Sun). Auroras occur when energetic particles precipitate along these field lines into the upper atmosphere. Increasing the amount of open magnetic flux enlarges the polar cap, shifting its boundary - and with it the auroral oval - to lower latitudes. In this paper, we use a simple dipole model to estimate the flux increase required for auroras to be visible from the equator.

Dipole Model

We take the Earth's magnetic field to be an aligned dipole of moment M ($= 7.94 \times 10^{15}$ T m^3) [1]. On the Earth's surface ($r = R_E$), the radial component is

$$B_r(R_E, \theta) = \frac{2M\cos\theta}{R_E^3}, \quad (1)$$

as shown in [2], where θ is colatitude (0° at the pole, 90° at the equator).

The polar cap is modelled as a circular region extending from $\theta = 0$ to $\theta = \theta_{\rm pc}$ where $\theta_{\rm pc}$ is the colatitude of the polar cap. The total open flux is then

$$\Phi(\theta_{\rm pc}) = \int_0^{2\pi}\int_0^{\theta_{\rm pc}} B_r R_E^2 \sin\theta \, d\theta \, d\phi. \quad (2)$$

Performing the integral yields the standard dipole result

$$\Phi(\theta_{\rm pc}) = \frac{2\pi M}{R_E}\sin^2\theta_{\rm pc}, \quad (3)$$

which allows us to plot the open flux as a function of polar cap boundary latitude.

Visibility Criterion

For typical quiet-time conditions, the polar cap boundary lies at a magnetic latitude of $\sim 75°$ [3], corresponding to $\theta_{\rm pc} \approx 15°$.

Aurora visible from the equator do not require the polar cap to extend all the way to $\theta = 90°$. Auroral emissions occur at altitudes of roughly 100 - 200 km [4]. The line of sight to the aurora forms a right angled triangle with the Earth's radius (assuming the observer's line of sight is tangential to the Earth's surface), so by simple geometry:

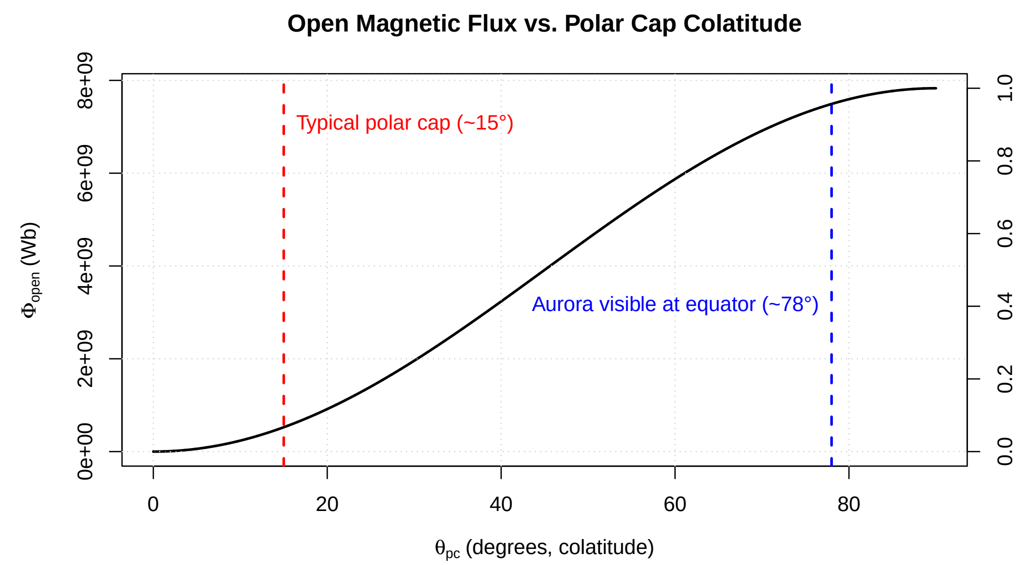

Figure 1: Open magnetic flux ϕ versus polar cap colatitude θ. The normalised flux ($\frac{sin^2\theta_{pc}}{sin^2\theta_{max}}$) is also shown on the right y-axis for easier comparison. The red line corresponds to the typical quiet-time boundary ($\theta = 15°$). The blue line corresponds to the approximate boundary required for auroral visibility at the equator ($\theta = 78°$).

$$d = \sqrt{(R_E + h)^2 - R_E^2} \approx \sqrt{2R_E h}. \quad (4)$$

For $h = 150$ km (taking the average auroral height [4], and assuming the h^2 term is negligible), this gives $d = 1.4 \times 10^3$ km, corresponding to roughly 12° of latitude and a colatitude of:

$$\theta_{\text{vis}} \approx 78°. \quad (5)$$

Results

Figure 1 shows the relation between the open flux and polar cap latitude according to our model. Vertical lines mark the typical quiet-time boundary (15°) and the visibility threshold for equatorial observers (78°).

At these points, the ratio of the normalised flux at these points is as follows:

$$\frac{\Phi(78°)}{\Phi(15°)} \approx \frac{0.96}{0.067} \approx 14.3. \quad (6)$$

This factor represents a lower bound on the required increase in open flux.

Discussion and Conclusions

The simple model shown in Figure 1 highlights the open-flux dependence on polar cap size. Pushing the boundary from 15° to 78° co-latitude increases open flux by a factor of 14.3 according to the model.

Because the dipole model ignores solar-wind compression, magnetotail stretching, ring-current weakening, and more realistic field-line mapping, this factor should be interpreted as a strict minimum. Realistic global simulations and empirical storm models suggest that the actual increase in open flux during extreme events is likely greater. Nevertheless, the simple dipole picture suffices to show that equatorial aurora imply a dramatic expansion of the region of open field lines, consistent with an extremely disturbed magnetosphere.

References

[1] Encyclopaedia Britannica. Dipolar field. https://www.britannica.com/science/

geomagnetic-field/Dipolar-field, 2025. Accessed: [25/11/2025].

[2] Margaret G. Kivelson and Christopher T. Russell. *Introduction to Space Physics*. Cambridge University Press, Cambridge, 1995.

[3] NOAA Space Weather Prediction Center. Aurora. https://www.swpc.noaa.gov/phenomena/aurora. Accessed: [25/11/2025].

[4] NASA. Auroras. https://science.nasa.gov/sun/auroras/, 2023. Accessed: [25/11/2025].