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A4 9 The Barrels

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Abstract

We model Pascal's barrel through the hydrostatic paradox. A 10 m long, 1 cm wide tube is fixed to a sealed 225 L barrel; adding 0.785 kg of water raises the head to produce 98.1 kPa gauge pressure at the lid. This pressure yields a 25.9 kN upward force on the lid despite the small added mass, illustrating that pressure depends on height and density, not container volume. The paradox is resolved by force balance: the lid exerts an equal 25.9 kN reaction on the fluid, so the barrel base must carry both the total fluid weight of 2.2 kN and this additional downward load. The results align with hydrostatics and validate the model.

Introduction

The principles of fluid statics, established by Blaise Pascal, include the counterintuitive concept known as the hydrostatic paradox. This principle states that the pressure exerted by a fluid at a certain depth depends only on the vertical height of the fluid column above it, regardless of the shape of the container or the total volume (and thus, weight) of the fluid [1].

A classic demonstration of this is Pascal's Barrel experiment [2]. In this scenario, a tall, narrow tube is inserted into the lid of a sealed, water-filled wooden barrel. By adding a very small volume of water into the tube, the pressure rises to a point where the barrel bursts.

This paper investigates the quantitative feasibility of this experiment. We aim to calculate the precise forces generated on the barrel's lid and resolve the apparent paradox of how a negligible mass of water can create barrel-failing force.

Theory

For a fluid at rest in a uniform gravitational field $\mathbf{g}$, the pressure field satisfies [3]

$$\nabla P = \rho\,\mathbf{g}. \tag{1}$$

If gravity acts in the negative z-direction ($\mathbf{g} = -g\,\hat{\mathbf{z}}$) and the free surface is at $z = h$, integration gives

$$P(z) = P_0 + \rho g(h - z), \tag{2}$$

where P_0 is the atmospheric pressure at the surface and $P(z)$ is the absolute pressure at depth z.

At the bottom of the container ($z = 0$) the absolute pressure is

$$P_b = P_0 + \rho g h. \tag{3}$$

The gauge pressure, defined as the pressure above atmosphere, is therefore

$$P_g = P_b - P_0 = \rho g h. \tag{4}$$

The net downward force F_b exerted on a flat horizontal base of area A is

$$F_b = P_g\,A = \rho g h A. \tag{5}$$

This result, $F_b = \rho g h A$, is the core of the paradox: the base force depends on h and A, but not on the total fluid volume V or weight $W = \rho g V$.

Resolving this requires a vertical force balance: the upward base force supports the fluid weight plus the vertical component of the sidewall force, F_{wv}. In inward-tapering vessels, walls push down on the fluid ($F_{wv} < 0$).

$$F_{\text{base,up}} + F_{wv} = W$$

By Newton's third law, the downward force on the base equals the upward reaction, $F_b = F_{\text{base, up}}$. For a container that tapers inward, such as Pascal's Barrel, the walls push down on the fluid, so $F_{wv} < 0$. This means the force on the base, $F_b = W - F_{wv}$, is greater than the fluid's weight.

Model Analysis

This analysis models the hydrostatic paradox using a standard 225 L wine barrel. We assume the barrel is full of water and that the free surface is located at the top of the vertical tube. The barrel is treated as a cylinder sealed with a flat lid, into which a narrow vertical tube is inserted. The lid provides a downward normal load $P_g \times A_{\text{lid}}$ on the fluid. While real barrels may not physically taper, the model considers the effective cross-sectional area at the lid, where pressure from the tube is applied.

Model Parameters - Calculations assume $g = 9.81\,\text{m/s}^2$ and water density $\rho = 1000\,\text{kg/m}^3$ [4]. The barrel volume is $V_{\text{barrel}} = 0.225\,\text{m}^3$ (225 L). The lid diameter is 0.58 m ($r_{\text{lid}} = 0.29\,\text{m}$) [5], yielding an area $A_{\text{lid}} \approx 0.2642\,\text{m}^2$. The attached tube has a height $h = 10.0\,\text{m}$ and inner radius $r_{\text{tube}} = 0.005\,\text{m}$.

Analysis of Hydrostatic Force - First, the gauge pressure (P_g) exerted by the 10.0 m water column at the level of the lid is calculated using Equation (4), which gives 98.1 kPa

This pressure acts uniformly across the entire inner surface of the barrel lid, generating a total upward force (F_{lid}).

$$\begin{aligned} F_{\text{lid}} &= P_g \times A_{\text{lid}} \\ &\approx 25.919\,\text{kN}. \end{aligned}$$

This calculated force is then contrasted with the total weight of the fluid (W_{total}). The mass of the water in the tube (m_{tube}) is:

$$\begin{aligned} V_{\text{tube}} &= 7.854 \times 10^{-4}\,\text{m}^3. \\ m_{\text{tube}} &= \rho V_{\text{tube}} \approx 0.785\,\text{kg}. \end{aligned}$$

The total fluid mass is $m_{\text{total}} = m_{\text{barrel}} + m_{\text{tube}} = 225\,\text{kg} + 0.785\,\text{kg} = 225.785\,\text{kg}$. The total fluid weight (W_{total}) is 2,215 N.

Force Resolution - A significant discrepancy exists: the upward force on the lid (F_{lid}) is more than 11 times greater than the total fluid weight (W_{total}).

This paradox is resolved by a force balance on the entire fluid volume. In this model, the lid acts as the inward-tapering wall, pushing down on the fluid. By Newton's third law, this downward force ($F_{\text{lid, down}}$) is equal in magnitude to the upward bursting force exerted by the fluid on the lid:

$$F_{\text{lid, down}} = F_{\text{lid}} \approx 25.9\ kN$$

The total upward force from the barrel's base (F_{base}) must therefore balance both the fluid's weight and this large downward force from the lid:

$$\begin{aligned} F_{\text{base}} &= W_{\text{total}} + F_{\text{lid, down}} \\ &\approx 2.2\ kN + 25.9\ kN \approx 28.1\ kN \end{aligned}$$

This result resolves the paradox by accounting for the 25.9 kN downward reaction force from the lid.

Conclusion

The analysis demonstrates that 0.785 kg of water in a 10 m tube generates a 25.9 kN bursting force on the lid. This load is assumed to rupture a standard barrel, subject to specific material limits. Force equilibrium resolves the paradox: the base supports the fluid weight (2.2 kN) plus the downward reaction from the lid (25.9 kN).

References

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