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A1 6 Fire In The (Variable) Hole

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Abstract

This paper lays a theoretical framework for applying shape memory alloys (SMA) to rocket nozzle design to maintain ideal expansion. We quantify the significant performance gains, measured by the Thrust Coefficient (C_T), by comparing a variable nozzle to a fixed nozzle. We apply this framework to the Atlas V 401 1st stage, tabulating the required exit area (A_e) and resulting C_T at key altitudes. The results establish the clear gains in efficiency and provide a first look into the requirements that future research must address.

Introduction

Nozzle shape is critical to rocket mission efficiency as it dictates the exhaust gas flow. In general, there are 3 types of exhaust shapes, under-expanded, over-expanded and ideal expansion. Ideal expansion occurs at equilibrium of the nozzle exit pressure (P_e) to the ambient atmospheric pressure (P_a) [1]. As a rocket ascends, P_a decays exponentially [2], necessitating a continuous increase in the nozzle's exit area (A_e) to follow this pressure drop. Fixed geometry de Laval nozzles [3] are only optimal at one altitude. This results in significant thrust losses over the course of a mission.

Multiple attempts at tackling this have been considered, such as the aerospike nozzle [4]; however, the added complexity, cost, and mass result in them being mostly unviable. This paper considers shape memory alloys (SMAs) as an actuation method and uses the governing equations for the ideal exhaust area. SMAs are any metal alloy that can be deformed but will return to its original shape after being heated above a certain temperature [5]. This offers a low mass way of changing nozzle geometry, allowing for extended time at ideal expansion and thus greater efficiency. The only input needed would be through thermal interactions, an abundant resource in this high-temperature environment.

Analysis

The following analysis assumes one-dimensional, isentropic, compressible flow in a de Laval nozzle (the standard nozzle design), as well as assuming a standard atmospheric model [2]. For an exhaust gas with specific heat ratio (γ), the pressure at the nozzle exit is related to the chamber pressure (P_c) and the exit Mach number (M_e) by the isentropic pressure relation [6]:

$$\frac{P_e}{P_c} = \left(1 + \frac{(\gamma - 1)}{2} M_e^2\right)^{-\frac{\gamma}{\gamma-1}}. \quad (1)$$

Rearranged in terms of the Mach number:

$$M_e = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_c}{P_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}. \quad (2)$$

The expansion ratio (ϵ) is a ratio of the fixed throat area (A_t), to exhaust area (A_e) and is used to find the optimal A_e at different altitudes, given M_e and γ [7]. To implement this, the area-Mach relation [7] is used:

$$\frac{A_e}{A_t} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}. \quad (3)$$

Ambient pressure ($P_a(h)$), at a given altitude (h), is modelled by the barometric formula [2]:

$$P_a(h) = P_0 e^{-\frac{h}{H}}. \quad (4)$$

Here, we use pressure (P_0) of 101.325 kPa, and atmospheric scale height (H) of 8.5 km [8]. Under ideal expansion, exit pressure and ambient pressure are equivalent. This is applied to the Atlas V 401 1st stage rocket with parameters of; chamber pressure 27.6 MPa, specific heat ratio of 1.22, and a nozzle throat area of 0.087 m^2 [7]. To quantify the efficiency, we use the Thrust Coefficient (C_T), the sum of the momentum thrust (C_F) and the pressure thrust:

$$C_T = C_F + \epsilon\frac{(P_e - P_a)}{P_c}. \quad (5)$$

Since P_e and P_a are equal the second term becomes zero. Thus C_T is equal to C_F. For the ideal case, P_e is replaced with P_a below [9]:

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_a}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}. \quad (6)$$

Table 1: Calculated ideal nozzle parameters.

Alt., h **(km)**	**Amb. Press.,** P_a **(kPa)**	**Mach,** M_e	**Ideal Exit Area,** A_e **(m^2)**	**Thrust Coeff.** C_T
0	101.3	3.99	2.13	1.72
5	56.3	4.32	3.32	1.78
10	31.2	4.67	5.28	1.83
20	9.63	5.40	13.4	1.91
40	0.916	7.02	86.9	2.01

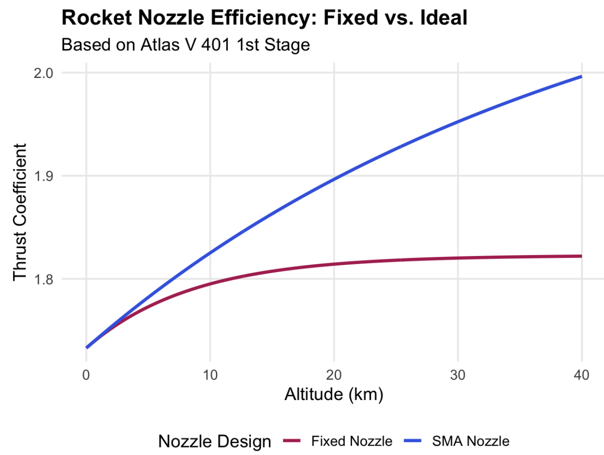

Figure 1: Rocket Nozzle Efficiency: Fixed vs. Ideal

Discussion

Figure 1 illustrates the benefit of the SMA nozzle. The Fixed Nozzle's C_T plateaus around 1.82, representing significant thrust loss at altitude. In contrast, the variable SMA Nozzle continuously increases, reaching 1.91 by 20 km (Table 1) - a near 10% gain. This continues to rise with altitude, highlighting the clear advantage of variable geometry.

However, Table 1 also reveals the immense challenge. To achieve this benefit, A_e must increase from 2.13 m^2 to 13.4 m^2 in the first 20 km, and then to 86.9 m^2 by 40 km. This non-linear expansion, would be a major material science and engineering hurdle for any SMA.

Conclusion

This paper established a framework for maintaining ideal expansion through SMAs, quantifying the efficiency gains (Figure 1) over fixed nozzles. This demonstrates teh potential for increased payload capacity or fuel savings. However, the analysis also highlights the core challenge: the extreme, non-linear are change required (Table 1). Future work should focus on the material science of a viable SMA capable of such deformations, and on the control systems needed to manage this variable rate.

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