

# Journal of Physics Special Topics

An undergraduate physics journal

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## A4 6 Frozen's Ice Generation

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November 15, 2025

### Abstract

From the movie *Frozen*, we investigate if the radial icicle growth is due to environmental conditions or Elsa's powers. We calculated the heat transfer coefficient required for the rate of radial growth to be  $\approx 9.76 \times 10^6 \text{ W m}^{-2} \text{ K}^{-1}$ . The coefficient based on the environmental conditions is in the range of  $\approx 1.11$  to  $7.5 \times 10^{-2} \text{ W m}^{-2} \text{ K}^{-1}$ . Due to a difference of six to eight orders in magnitude, we can conclude that the growth rate is not solely due to the environment.

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### Introduction

In the 2013 Disney movie *Frozen*, the Queen of Arendelle, Elsa, has displayed a magical ability to generate ice. She demonstrated her abilities in the film by creating a large ice chandelier [1].

This paper compares the mean heat transfer coefficient of the environmental conditions to the calculated value based on Elsa's powers to determine if she is the cause of the phenomena. We illustrate this point by using the movie's environmental conditions, based on Norwegian winters and the rate of icicle growth by treating the chandelier as a large icicle.

### Theory behind Icicle Formation

Icicle formations occur from a liquid source at the base of an already present ice formation, creating a fluid film that flows towards the tip due to drag or gravity. As long as the water flow is large enough, the icicle will experience a uniform coating. The freezing of the water causes the icicle to grow, which releases the latent heat of fusion from the water/ice interface [2].

### Radial Icicle Growth Calculations

**Elsa's icicle growth rate** - We want to find the mean heat transfer coefficient of the icicle walls formed by Elsa. We find the value using the growth rate of the ice chandelier and treat it as a large icicle.

The equation to calculate the rate of mean diameter growth of an icicle is given based on the analytical icicle growth model proposed by Makkonen [2]:

$$\frac{dD}{dt} = \frac{-ht + h \frac{0.622L_e}{c_p p_a} \{e(0^\circ\text{C}) - Re(t)\} - \sigma a t}{\frac{1}{2}\rho_a L_f (1 - \lambda)}. \quad (1)$$

The assumed diameter growth rate of the ice chandelier,  $\frac{dD}{dt}$ , is set as  $\approx 0.8 \text{ m s}^{-1}$  based on the movie's dimensions and song time [1]. The radiation parameter,  $a$ , is  $\approx 8.1 \times 10^7 \text{ K}^3$  [2]. The air temperature,  $t$ , is given as  $\approx -5^\circ\text{C}$  and the relative humidity,  $R$ , is taken as  $\approx 0.86$  for Norway in winter [3]. The atmospheric air pressure,  $p_a$ , is  $\approx 1.0 \times 10^5 \text{ Pa}$  [4]. The latent heat of evaporation of water,  $L_e$ , is  $\approx 2.0 \times 10^6 \text{ J kg}^{-1}$  [5], while the latent heat of fusion,  $L_f$ , is  $\approx 3.0 \times 10^5 \text{ J kg}^{-1}$  [6]. The specific heat of air,

$c_p$ , is  $\approx 1000 \text{ J kg}^{-1} \text{ K}^{-1}$  [7]. The saturation vapour pressure at  $0^\circ\text{C}$ ,  $e(0^\circ\text{C})$ , is  $\approx 600 \text{ Pa}$ ; and at  $-5^\circ\text{C}$ ,  $e(t)$ , is  $\approx 400 \text{ Pa}$ . These represent the maximum water vapour the air can hold at a specific temperature based on the improved Magnus formula for water [8]. The density of an icicle's wall,  $\rho_a$ , is  $\approx 900 \text{ kg m}^{-3}$  [9]. The liquid fraction of ice,  $\lambda$ , (the fraction of the icicle wall's volume that is trapped as water) is 0.26 [2].

By rearranging Equation 1, we can find the heat transfer coefficient from:

$$h = \frac{\left(\frac{dD}{dt}\right) \left(\frac{1}{2}\rho_a L_f(1-\lambda)\right) + \sigma at}{\frac{0.622L_e}{c_p \rho_a} \{e(0^\circ\text{C}) - Re(t)\} - t}.$$

Using these values, we calculate  $h$ , the heat transfer coefficient of the icicle's walls, to be  $\approx 9.76 \times 10^6 \text{ W m}^{-2} \text{ K}^{-1}$ .

**Environmental icicle growth rate** - We can now calculate the mean heat transfer coefficient of the walls of the icicle,  $h$ , from the formula [2]:

$$h = \frac{k_a Nu_w}{L}. \quad (2)$$

The thermal conductivity of air,  $k_a$ , is  $\approx 26 \times 10^{-3} \text{ W m}^{-1} \text{ K}^{-1}$  at  $-5^\circ\text{C}$  [10]. The length of the icicle,  $L$ , is taken as  $\approx 1.0 \text{ m}$  based on the approximate length of the chandelier [1].

The variable  $Nu_w$  is the Nusselt number, which is a dynamical variable showing the ratio of convective to conductive heat transfer. Nusselt's number depends on the higher number from free or forced convection [11].

Since the chandelier is created in an enclosed room without significant air currents, we assume free convection is the dominant mode of heat transfer, rather than forced convection. For free convection, we would need to find a preliminary Nusselt number  $Nu_{W,L}^N$  [2]:

$$Nu_{W,L}^N = 0.478 Gr^{0.25} \quad (3)$$

where  $Gr_t$  is the Grashof number. This number describes the ratio of a fluid's buoyancy to its viscosity [12] given as [2]:

$$Gr = \frac{gd^3(0^\circ\text{C} - t)}{v^2(t + 273.15^\circ\text{C})} \quad (4)$$

where the kinematic viscosity of air,  $v$ , is  $\approx 1.3 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$  at  $-5^\circ\text{C}$  [13].  $d$ , is the diameter of the pendant drop which we can say is the initial mean diameter of the icicle  $\approx 0.01 \text{ m}$ . We calculate a Grashof number of 1082.3 which gives a preliminary Nusselt number of 2.74.

To find the Nusselt number corrected for curvature we found the curvature correction [2]:

$$S = \frac{2L}{DNu_{W,L}^N} \quad (5)$$

where,  $D$ , is the mean diameter of the icicle which will vary from  $\approx 0.01 \text{ m}$  to  $\approx 6 \text{ m}$  as the chandelier grows. The correction value,  $S$ , has a calculated range from 72.99 to 0.12.

For free convection, the Nusselt number corrected for curvature is calculated using the formula [2]:

$$Nu_w = \frac{0.9SNu_{W,L}^N}{\ln(1 + 0.9S)}. \quad (6)$$

Using the values calculated before we get a Nusselt number range,  $Nu_w$ , of 42.81 to 2.89. Using the calculated values, we found the mean heat transfer coefficient of the walls of the icicle,  $h_w$ , from Equation 2 being  $\approx 1.11 \text{ W m}^{-2} \text{ K}^{-1}$  to  $\approx 7.5 \times 10^{-2} \text{ W m}^{-2} \text{ K}^{-1}$  respectively.

## Conclusion

This paper examines the heat transfer coefficient of the icicle walls based on the radial rate of icicle growth from Elsa, which we calculate to be  $\approx 9.76 \times 10^6 \text{ W m}^{-2} \text{ K}^{-1}$ . We can compare the heat transfer coefficient to what is expected based on the assumed environmental condition, which is from  $\approx 1.11 \text{ W m}^{-2} \text{ K}^{-1}$  to  $\approx 7.5 \times 10^{-2} \text{ W m}^{-2} \text{ K}^{-1}$ . Comparing the values gives a difference of six to eight orders of magnitude.

While our calculations rely on assumptions about the environmental parameters, the vast difference in coefficients strongly suggests Elsa artificially increases the heat transfer coefficient of the icicle walls.

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