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## P5 2 The Laziest Student and Their Relativistic Pen

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### Abstract

In this paper we will examine the hypothetical problem of how late a student could leave their studying for an entire MPhys degree at the University of Leicester, if they were in possession of a pen able to write at relativistic speeds. After accounting for time dilation, we find that a pen moving just fast enough for relativistic effects to be apparent ( $v = 0.5c$ ), would be observed to take an estimated  $7.31 \times 10^{-4}$  s to write notes on the entire degree.

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### Introduction

In 1905, Albert Einstein published *The Electrodynamics of Moving Bodies* [1], introducing the world to special relativity. This new theory described how the behavior of objects moving at velocities approaching the speed of light cannot be described using only Newtonian mechanics and instead exhibit relativistic effects. The effect we will primarily be focusing on in this paper is time dilation, in which the faster an object moves, the slower their clock ticks compared to a stationary object.

### Method

The first step of this problem is to determine what is meant precisely by “relativistic speeds”, that is, how fast an object needs to move for relativistic effects to be considered. We can do this by analyzing a graph of how relativistic momentum differs with Newtonian momentum at high velocities and seeing at what point they diverge from one another.

The Newtonian momentum of an object comes from the equation:

$$p = mv \tag{1}$$

And the relativistic momentum of an object can be found from:

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} \tag{2}$$

Both functions can be plotted on a momentum vs velocity graph, showing velocities from zero to  $c$ , so that we can see the velocity at which these two functions diverge.

The next step was to figure out how far the pen will need to move to make notes on all of the lectures in the 4-year MPhys course. This was done by randomly selecting two lectures in a taught module for each year of the course, and then utilising Microsoft Word, to count the number of words in each lecture. After an average word count per lecture is estimated, this can be multiplied by 576 to get an estimate for the total number of words across all lectures in the course. The 576 factor is calculated on the assumption of 8 lectures per unit, with 3 units per module and 24 taught modules across the degree. One randomly selected lecture was then copied into a word counter to find the average length of each word, using the given word count as well as

character count. Multiplying the average word length by the total word estimate, gives us an estimate for the total number of letters that a student would need to write to make notes on the whole course.

We then modelled each letter to be a 4 mm × 4 mm square, half the height of the spacing on a standard piece of lined paper. Therefore, the student would need to move the pen 16mm to draw out each letter, plus 1mm to move the pen upwards off the page, 1mm of spacing between the letters, and finally 1mm back onto the page to begin writing the next letter, resulting in a total of  $1.9 \times 10^{-2}$  m of pen movement per letter. We can multiply this by our total letter count to get the total distance the pen has to move in its own reference frame. In the reference frame of the pen, we can calculate the time taken for the pen to travel this distance assuming a constant speed of  $0.5c$  using the classical:

$$v = \frac{s}{t} \quad (3)$$

However, in the reference frame of the observer we need to take into account time dilation. Therefore, we use the time dilation equation:

$$\Delta t = \Delta t' \times \gamma \quad (4)$$

where  $\Delta t$  is a measured time interval in the frame of the moving object,  $\Delta t'$  is the time measured by a stationary observer and  $\gamma$  is the lorentz factor. This is used to tell us how long a stationary observer will view the pen moving for and thus how long it would take to produce notes on the entire degree with the relativistic pen.

## Results and Conclusion

If we examine the graph shown in Figure 1 [2], we can see that at around the  $0.5c$  mark is when we begin to see noticeable differences between relativistic and Newtonian momentum. Therefore, we can assume that our pen is moving at  $v = 0.5c$ .

From the data we collected the average word count was found to be 1,461 words per lecture, with an average word length being 6 letters. If we

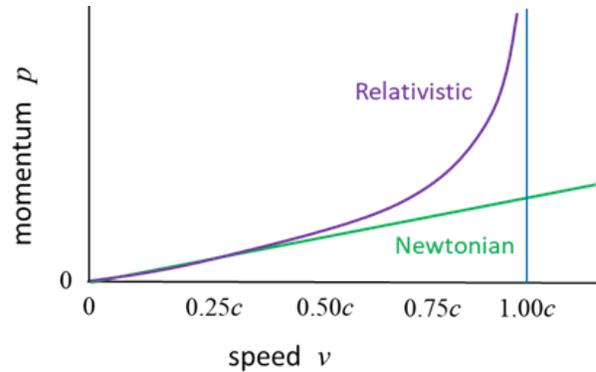


Figure 1: A graph showing the changes in relativistic and Newtonian momentum as velocity increases from 0 up to  $c$ .

multiply our two values together, we reach a total of approximately 5,000,000 individual letters across all lecture slides that our student needs to write up. It should be noted that in reality, most students tend not to write up entire lecture slides word for word, meaning our estimate could be a slight overshoot. Our value has the pen having to move a total distance of 95,000 m. We now have to assume that the pen is moving at a constant speed of  $0.5c$  which is unrealistic, as the pen would accelerate as it changes directions leading to differing levels of relativistic effects at different stages of the drawing process of each letter. However, if we assume an average speed of  $0.5c$  the final times should be the same. We can calculate that from the reference frame of the pen, the task would take  $6.3 \times 10^{-4}$  s, however when time dilation is accounted for, we find that a stationary external observer times the task to take  $7.31 \times 10^{-4}$  s.

## References

- [1] [www.fourmilab.ch/etexts/einstein/specrel1/www/](http://www.fourmilab.ch/etexts/einstein/specrel1/www/) [Accessed 10th October 2025]
- [2] [d-arora.github.io/VisualPhysics/mod72/mod72F.htm](https://d-arora.github.io/VisualPhysics/mod72/mod72F.htm) [Accessed 10th October 2025]