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A5 8 If you liked it, you should have put a Neptune on it

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Abstract

Neptune experiences pressure sufficient to produce diamonds and this would occur in the layer of Neptune called the icy layer. The distance travelled by the carbon atoms, from the boundary of the atmosphere to the icy layer to form a diamond is found to be 4329 km. The mass of diamonds that could be formed in the entire planet was calculated to be 6.99×10^{23} kg. If this amount of diamond was sold on Earth, it would approximately cost £1.05×10^{31}.

Introduction

Previously discussed in the article 'A5 5 Jupiter is a girl's best friend' [1], a physicist travels to Jupiter to investigate the mass of diamonds they can find and sell on Earth. The physicist now wonders what the mass of diamonds is on Neptune.

Diamonds are formed in Neptune by the following process [2]; methane in the icy layer (found at a depth of 3000 km) experiences a large amount of pressure and heat, which makes methane chemically active. The methane separates into its constituent atoms: 1 part carbon and 4 parts hydrogen. The carbon atom then travels down the layers of Neptune, towards the core, as shown in Figure 1, and is compressed into a diamond.

The assumptions made are as follows; methane is distributed evenly throughout the layers of Neptune's atmosphere; when methane is split into carbon and hydrogen, the atoms will not recombine, all carbon atoms will turn into a diamond, and Neptune's gravity is constant.

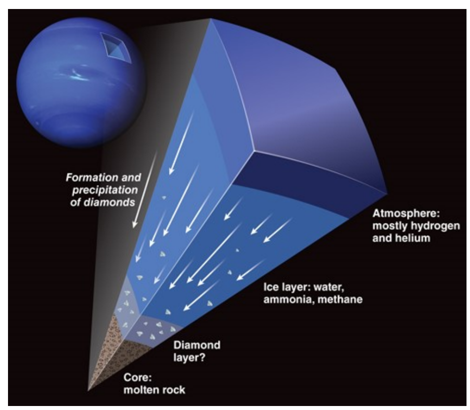

Figure 1: Neptune and its layers. A brief description of each layers chemical make-up and the diamond making process [2].

Under pressure - Part 2

The physicist needs to know the depth at which carbon will experience sufficient pressure to produce diamonds. Following the method outlined in [1], we start with the hydrostatic equilibrium equation that can be used to calculate the depth. This is stated below [3]:

$$\frac{\delta p}{\delta z} = -\rho g \tag{1}$$

Where δz is the change in depth (m) in a gravitational field, δp (Pa) is the change in pressure and g is the acceleration due to gravity, which is 11.27 ms^{-2} [4] on Neptune. The equation of state for an ideal gas, is as follows [3]:

$$p = \rho R_g T \tag{2}$$

Where p is the pressure (Pa), ρ represents the density (kg m^{-3}) of the gas at a given point, T represents the temperature (K). The specific gas constant is $R_g = R/\mu$, and we will be calculating this for hydrogen as it makes up 80% of Neptune's atmosphere [5]. It is assumed that the element is equally distributed throughout Neptune. R is the universal gas constant, and is 8.31 J mol^{-1} K $^{-1}$ [6] and μ represents the mass of one mole of a gas which is 2.016×10^{-3} kg mol^{-1} [6]. Combining equations 1 and 2 to remove density and then integrating:

$$p = p_0 e^{-z/H} \tag{3}$$

Where $H = R_g T/g = 692$ km. H is the pressure scale height, R_g is found to be 4124.26 J kg^{-1} K^{-1} and T is 1873.15 K (the temperature needed to produce a diamond) [7]. We rearrange equation 3 for z:

$$-z = \ln(\frac{p}{p_0})H \tag{4}$$

Where p is the pressure at the boundary of the icy layer and the atmosphere, which is 10 MPa [8]. p_0 is the pressure needed to form a diamond, 5.17×10^9 Pa [7]. The variables are entered into equation 4, and $z = 4392$ km. This depth is located in the icy layer, which is 17,500 km thick. [2].

Show me the money! - Part 2

Now that the depth at which the diamonds will form has been found, their mass can be calculated. 1.5% of Neptune's atmosphere is made up of methane [5]. The percentage of methane located in the icy layer is 0.91% with a volume of 3.86×10^{22} m^3 and the mass is 9.32×10^{23} kg of methane molecules [4]. Carbon makes up 75% of the mass of methane, so the mass of carbon is 6.99×10^{23} kg. To calculate the carat of a diamond using the carbon mass, divide by 0.2 g [9]. Carbon produces 3.50×10^{27} carats and 1 carat costs £3,000 [9]. Therefore, the price of diamonds would be £1.05×10^{31}.

Conclusion

If the physicist travelled to Neptune rather than Jupiter, the sale of the diamonds would be one point zero five decillion pounds. The disadvantages are that Neptune is further away and less missions have been carried out, compared to Jupiter. This means the physicist can plan their trip to Jupiter with more information. An advantage of travelling to Neptune is the physicist wouldn't have to avoid lighting strikes, (seen on Jupiter and discussed in A5 5 [1]) making it a safer trip overall.

References

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