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Abstract

In this paper we investigate how the relativistic Doppler effect can change the observed colours of sand in hourglasses. We calculate the velocity to obtain each colour in a rainbow, with wavelengths smaller than yellow traveling towards the observer, and greater wavelengths traveling away. The time difference due to time dilation and the effects of varying aperture size were also investigated.

Introduction

Hourglasses have been used for centuries as a way to measure the flow of time. By assuming yellow sand flows through a 30 second hourglass, we investigate how the relativistic Doppler effect can change the perceived colour of the sand in an hourglass to any colour in a rainbow, and the changes in duration of the hourglass at these high speeds due to time dilation. Additionally, we look at changing the aperture size of the hourglass to offset this time difference.

Theory

At relativistic speeds, the colour of light seen from an object changes due to the relativistic Doppler effect, given by [1]:

$$\lambda_{obs} = \lambda_r \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \tag{1}$$

where λ_{obs} is the observed wavelength, λ_r is the wavelength in the rest frame, v is the velocity of the hourglass, and c is the speed of light. By rearranging this equation, we obtain:

$$v = c\frac{(\frac{\lambda_{obs}}{\lambda_r})^2 - 1}{(\frac{\lambda_{obs}}{\lambda_r})^2 + 1} \tag{2}$$

We can now calculate the velocity at which an hourglass would appear a certain colour, given the initial wavelength of light in the rest frame.

From this, we calculate the time difference for an hourglass viewed by a stationary observer, using time dilation [2]:

$$t = \frac{t_r}{\sqrt{1-(\frac{v}{c})^2}} \tag{3}$$

where t is the time in the observer's frame and t_r is the time in the rest frame.

At relativistic speeds, the sand timers will appear to flow more slowly than in their rest frames. If we want every sand timer to last the same amount of time, we adjust the flow rate of the sand accordingly. A simple equation for volume flow rate is [3]:

$$Q = vA \tag{4}$$

where Q is volumetric flow rate, v is speed of flow, and A is cross-sectional area. In order for any hourglass to last the same amount of time, the observed volume flow rate must remain constant. Because the speed of flow is inversely proportional to time, this equation can be rewritten as:

$$A \propto A_r \frac{t}{t_r} \quad (5)$$

where A_r is the cross sectional area of the yellow hourglass at rest.

Discussion

We take our base hourglass to be a 30 second hourglass, chosen arbitrarily, with bright yellow sand, wavelength $\lambda = 580$ nm [4]. We have modeled each colour in a rainbow, and, using the theory discussed above, calculated the velocities at which each timer would be traveling, and the time difference each hourglass would appear to have because of time dilation:

Colour	λ (nm)	v ($10^7 m/s$)	δt (s)
Violet	415	-9.68	1.70
Indigo	430	-8.71	1.35
Blue	475	-5.91	0.60
Green	535	-2.42	0.10
Yellow	580	0.00	0.00
Orange	605	1.26	0.03
Red	685	4.94	0.42

Table 1: Table showing wavelength, velocity and time difference for each colour of sand.

We then plotted this information:

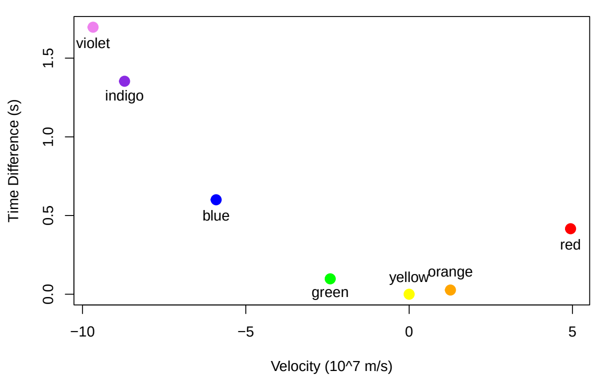

Figure 1: Graph showing the colour of each sand timer, the velocity at which it travels, and the time difference due to this velocity.

To interpret this information, we look at the hourglass we want to appear violet. In its rest frame, it is the same as any other hourglass used, with bright yellow sand and a duration of 30 seconds. In order for it to appear violet to an observer, the hourglass must be traveling at a relativistic velocity, calculated using equation (2) as -9.68×10^7 m/s. This is a negative velocity, which means that the hourglass must be traveling at this speed towards the observer.

From equation (5), we can see that observed area is directly proportional to observed duration. This means that if we wanted the hourglass to last exactly 30 seconds to an observer seeing it as violet, the area could be increased by the same proportion as the duration increase. if the yellow hourglass had a cross-sectional area of 0.5 cm^2, for example, the one appearing violet would need an area of 0.528 cm^2 in order to appear to have a duration of 30 seconds in the observer's frame.

Conclusion

Our results show that, in order to observe a rainbow of hourglasses, each hourglass must be traveling at relativistic velocities. Hourglasses with wavelengths shorter than yellow are traveling towards the observer, and longer wavelengths are traveling away from the observer. Additionally, for each hourglass other than the stationary yellow, the duration of the hourglass increases proportionally to the square of the velocity.

References

[1] `http://spiff.rit.edu/classes/phys314/lectures/doppler/doppler.html` [Accessed December 11, 2024]

[2] `https://www.phy.olemiss.edu/HEP/quarknet/time.html` [Accessed December 11, 2024]

[3] `https://www.lmnoeng.com/flowrate.php` [Accessed December 11, 2024]

[4] `https://www.lumitex.com/blog/visible-light-spectrum` [Accessed December 11, 2024]