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P4_5 33 Gigawatts to Oblivion: The Explosion That Changed Nuclear History

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Abstract
Reactor 4 at the Chernobyl Nuclear Power Plant experienced a catastrophic power surge on April 26, 1986, caused by a sudden reactivity increase due to the graphite tips of control rods, a positive void coefficient, and steam generation. This paper models the exponential power escalation using point kinetics, finding that the reactor's power surged from 200 MW to over 33 GW in just 4 seconds and would have reached approximately 70 TW within 10 seconds. The analysis highlights the rapid dynamics of the runaway reaction that occured in the reactor.

Introduction

On April 26, 1986, the Chernobyl Nuclear Power Plant experienced one of history's worst nuclear disasters. Reactor 4, an RBMK-1000 graphite-moderated, water-cooled reactor [1], catastrophically failed during a safety test due to design flaws and operator errors. Operating at just 200 MW [2], far below safe levels, removing most control rods eliminated negative reactivity margins. Combined with a positive void coefficient and xenon decay, this caused an exponential power surge. Activation of the AZ-5 shutdown button, exacerbated by graphite-tipped control rods, drove power to an estimated 33 GW [3], more than ten times the design limit. By modelling the runaway reaction using point kinetics, this paper explores the theoretical maximum power the reactor could have achieved in the absence of structural failure.

Theoretical Power and Point Kinetics

To understand the relation of this sudden power surge, the point kinetics is needed to understand the neutron effects. The point kinetic equation is a system of differential equations that model the time-dependent behaviour of nuclear reactor, which is given by [4]:

$$\frac{dn(t)}{dt} = \underbrace{\frac{\rho(t)-\beta}{\Lambda}n(t)}_{prompt\ neutrons} + \underbrace{\Sigma_{i=1}^{6}\lambda_i C_i(t)}_{delayed\ neutrons} \tag{1}$$

Since the reaction is a runaway reaction, the prompt neutrons would be dominating, hence the effect of delayed neutron in equation 1 is omitted. Equation 1 then becomes:

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda} \cdot n(t) \tag{2}$$

where $\rho(t)$ is the reactivity which is a measure of how far the reactor is from criticality, in this case of supercriticality, $\rho > 1$, β is the fraction of delayed neutrons, typically ranging from 0.0022 to 0.007 [5] and Λ is the average time between the release of a neutron from a fission event and its absorption or escape which is typically very small [6]. Therefore, the final equation with α as the growth rate for the reaction is:

$$\frac{dn(t)}{dt} = \alpha n(t)\,,\ \alpha = \frac{\rho(t) - \beta}{\Lambda} \tag{3}$$

Solving the ordinary differential equation in equation 3, the relation for neutron population is found:

$$n(t) = n_0 e^{\alpha t} \tag{4}$$

Since, power P(t) is directly proportional to the rate of energy release from fission reactions, which in turn is proportional to the neutron population n(t), equation 4 can be written as:

$$P(t) = P_0 e^{\alpha t} \tag{5}$$

Where P(t) is the final power measured before the reactor exploded (33 GW), P_0 is the initial power of the reactor (200 MW), and t is the time taken for power to reach the maximum reading which can be estimated from the timeline to be 4 seconds [2]. The ideal condition for a reactor to be supercritical is, $\rho > 1$ and delayed neutron effect, β and average time Λ are really small, therefore negligible, hence, $\alpha > 1$.

Thus, solving equation 5 for α, it is found to be equal to 1.27 s^{-1}.

Equation 5 is now modelled (in log scale) to find the exponential power surge of the reactor.

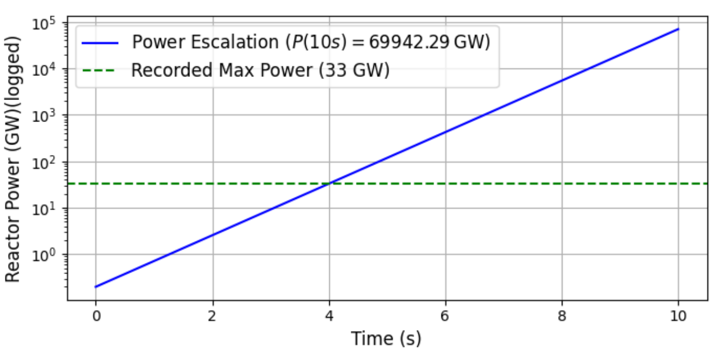

Figure 1: Theoretical power escalation in the Chernobyl Reactor 4 during the runaway event shown in a log plot, showing that power would've reach about 70 TW in 10 seconds.

Discussion and Conclusion

Using point kinetics, the Chernobyl Reactor 4 accident was modelled to reveal the exponential power growth, escalating from 200 MW to 33 GW in just four seconds, with a calculated growth rate of $\alpha \approx 1.27s^{-1}$. Model extrapolation predicts that, without structural failure, the reactor's power could have reached around 70 TW within 10 seconds, underscoring the reactor's extreme supercriticality. This power level emphasises the severity of the runaway reaction and the reactor's inability to mitigate such extreme conditions. While the model assumes constant reactivity (ρ) for simplicity, in reality, ρ dynamically increases due to the positive void coefficient. This reaction wouldn't continue infinitely as neutron poisoning would eventually stop the reaction.

References

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