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A5 5 Jupiter is a girl's best friend

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Abstract

The pressure within Jupiter's layers is sufficient to create diamonds, the process in which carbon combines to produce a diamond would occur in the layer of Jupiter called the molecular hydrogen layer. The distance travelled by the carbon atoms from the water clouds to form a diamond is found to be 2850 km. This depth is situated in the molecular hydrogen level. The mass of diamond that could be formed in the northern hemisphere of Jupiter before melting was found to be 7.32×10^{14} kg. If this amount of diamonds were sold on Earth it would approximately cost £1.10×10^{22}.

Introduction

Diamonds are known to be a girl's best friend but what if a physicist wants them all? A physicist travels to Jupiter to investigate the mass of diamonds they can find and sell on Earth.

Diamonds are formed on Jupiter by the following process; lightening strikes occur above the water clouds where methane is present (this being 50 km deep into Jupiter's atmosphere from the surface [1]). When the lightning strikes, methane molecules break down into their constituent atoms: 1 part carbon and 4 parts hydrogen [2]. The carbon atom will travel down the layers of Jupiter's atmosphere and experience immense force which will create graphite, this will then form diamonds and eventually melt when reaching the core.

The assumptions made are as follows; methane is distributed evenly throughout the layers of Jupiter's atmosphere, lightning strikes effect all of the methane in the northern hemisphere. When the methane is split apart into carbon and hydrogen they will not recombine and all of the carbon will be converted to diamond. We additionally assume that Jupiter's gravity is constant. The northern hemisphere will only be considered, because there are more frequent strikes in the north, seen in figure 1.

Figure 1: Lightning strikes occurring in the northern hemisphere of Jupiter. [3]

Under pressure

To collect as many diamonds as possible, the physicist needs to know the distance a carbon atom travels from the water clouds into the layers of Jupiter to form a diamond. To find the depth, the hydrostatic equilibrium equation can be used, which is stated below [5]:

$$\frac{\delta p}{\delta z} = -\rho g \tag{1}$$

Where δz represents depth (m) in a gravitational field, δp (Pa) represents change in pressure, ρ (kgm^{-3}) represents the density of the gas at a given point, and g is the acceleration due to gravity which is 24.79 ms^{-2} [4]. Using the equation of state for an ideal gas [5], as follows:

$$p = \rho R_g T \tag{2}$$

Where p represents pressure (Pa), T is temperature (K) and $R_g = R/\ \mu$. R_g is the specific gas constant, which will be the constant hydrogen as it constitutes 90% [6] of Jupiter's atmosphere. It is assumed that the element is equally distributed throughout Jupiter so R (the universal gas constant) is 8.31 J mol^{-1} K^{-1} [7] and μ is the mass of one mole of the gas which is 2.016×10^{-3} kg mol^{-1} [7]. Now combining equations 1 and 2 to eliminate density and integrating:

$$p = p_0 e^{-z/H} \tag{3}$$

Where $H = R_g T/g = 311$ km is the pressure scale height. R_g is calculated to be 4124.26 J kg^{-1} K^{-1} and T is 1873.15 K [8]. To find the depth where the pressure will be sufficient, we rearrange equation 3 for z:

$$-z = \ln(\frac{p}{p_0})H \tag{4}$$

Where p is the pressure at the water clouds which is 550 kPa [8] and p_0 is the pressure needed to form a diamond, 5.17×10^9 Pa [8]. The variables are inputted in equation 4, and $z \approx 2850$ km. This depth is located in the molecular hydrogen layer, and extends to a depth of 20,000 km within Jupiter's atmosphere [9].

Show me the money!

We now know where the diamonds are formed, but how much could there be? Only 0.15% [10] of the northern hemisphere is methane. The percentage of methane above the water clouds is 5.14×10^{-11}% of Jupiter's mass which is 9.76×10^{14} kg of methane molecules [5]. Carbon makes up 75% of the mass of these molecules, so the mass of carbon atoms is 7.32×10^{14} kg. To work out the carat of a diamond using the mass of carbon, you divide by 0.2 g [11]. The carbon produces 3.66×10^{18} carats and 1 carat on average costs £3000. Therefore, the price of the diamonds collected would be £ 1.10×10^{22}.

Conclusion

The physicist would make 11 sextillion pounds if they were able to bring back 7.32×10^{14} kg of diamond. In reality, how would the rich physicist bring back such a large mass?

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