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P2 4 Can We Get Much Higher?

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Abstract

In this paper, we discuss the minimum height a 11-year-old child would have to jump on a polypropylene trampoline in order to break the trampoline mat and fall through it. This height is calculated to be approximately 20.4 m, which would be fatal if attempted.

Introduction

Trampolining is a popular activity, enjoyed by children and adults alike, both recreationally and as a competitive sport. A modern trampoline is typically composed of a steel frame, a mat made from polypropylene, and springs also made of steel.

In this paper, we analyse the height an 11-year-old would need to jump from to cause the trampoline mat to break upon impact.

Theory

In order to find the maximum jumping height of the child, we first calculate the breaking strength of the trampoline mat. To do this we model a trampoline that consists of two layers [1] of thin polypropylene strands, criss-crossed to form a square mat. For simplicity, we disregard spring deformation and model the trampoline as a flat table with a polypropylene top, ignoring material mechanics.

Each strand has a cylindrical shape. To calculate the force an individual strand can withstand before breaking, we use the equation [2]:

$$F_{strand} = A\sigma_{max} \tag{1}$$

where F_{strand} is the maximum force, σ_{max} is the ultimate tensile stress, and A is the cross sectional area of a strand. From this, we can determine the maximum strength of the entire trampoline:

$$F_{max} = \frac{2LF_{strand}}{d} \times 10\% \tag{2}$$

where F_{max} is the maximum strength of the entire trampoline mat, L is the length of the trampoline, and d is the diameter of one strand. As the child will only make contact with a fraction of the mat when they jump on the trampoline, we will model the force as acting over an arbitrary value of 10% of the surface area of the mat.

To determine the acceleration at which the child slows down to a stop, we use the equation [3]:

$$F_{max} = \frac{\Delta p}{\Delta t} \tag{3}$$

where Δp is the change in momentum of the child, and Δt is the amount of time the change in momentum takes place in. In this scenario we are considering the point at which the child makes contact with the trampoline at full speed to when the velocity is 0 ms^{-1} at the bottom.

By assuming a value for the change in time, we are able to calculate the change in momentum of the child as they decelerate on the trampoline. The change in momentum is given by [3]:

$$\Delta p = m\Delta v \tag{4}$$

where m is the mass of the child, and Δv is the change in velocity. Rearranging equation (3) and substituting equation (4) the final velocity of the child is determined, assuming they come to a complete stop on the trampoline:

$$v = \frac{F\Delta t}{m} \tag{5}$$

Now that we have the final velocity of the child, we can use an equation of motion at constant acceleration to determine the height from which the child falls [3]:

$$s = \frac{v^2 - u^2}{2g} \tag{6}$$

where s is the minimum height the child needs to jump from to cause the mat to break, u is the initial velocity, v is the final velocity, and g is the acceleration due to gravity.

Analysis

For this paper, we use a square trampoline with a side length of 0.83 m, L [4]. We also assume the trampoline is positioned high enough so that it does not touch the ground, even when fully extended. Additionally, the child jumps from the center of the trampoline and all energy is conserved within the system.

The trampoline mat has an ultimate tensile strength, σ_{max}= 29 MPa [5], with one strand of polypropylene having a diameter $d = 5 \times 10^{-4} m$ [6] which in turn means that $A = 1.96 \times 10^{-7} m^2$. With these values we calculate $F_{strand} = 5.69$ N, using equation (1). This is used to find $F_{max} = 1.89 \times 10^3$ N, using equation eqrefe2.

To find the velocity, v, we assume that the child has a mass, m, of 35 kg [7] and the value of $\Delta t = 0.87 - 0.50 = 0.37$ s [8], assuming cycle time is invariant with height. Using equation (5), this leads to a velocity of $v = 20.0$ ms^{-1}. Substituting this into the equation of motion, assuming air resistance is negligible, that $u = 0$ ms^{-1}, and $g = 9.81$ ms^{-2}. Using equation (6), we obtain a final value of $s = 20.4$ m.

Conclusion

Our calculations show that the minimum height a 11-year-old child would need to jump from, to break the trampoline mat, is approximately 20.4 m. This is an unrealistic value, with other parts failing at much lower forces. This height is hazardous and would result in severe or fatal injuries, highlighting the strength of a polypropylene under typical use conditions. Further studies could examine the combined effects of spring deformation and dynamic forces, for a more realistic model.

References

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