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A1 3 Chaos of Meteorology

A. N. Tasyaka, K. Bujdoso, M. R. Stentiford and P. Holmes

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

In this paper, we investigated the Lorenz system; a scientific model in chaos theory - where changes to the initial parameters of a system greatly affects the resulting behaviour. The simulation was performed twice, using different two values of σ, 5.06 and 0.506. The subsequent graphs show the difference in σ, with the larger value showing more chaotic characteristics; having pronounced oscillations and repeated loops spiralling, whilst the smaller value followed a simple, predictable path. These results proved that a small change can have an enormous effect in a chaotic system.

Introduction

The 'Butterfly Effect' is a well-known phrase which refers to a small perturbation leading to unpredictable outcomes within the system [1]. This is derived from the work of Edward Lorenz, a meteorologist turned mathematician, who was interested in the prediction of weather through nonlinear systems. In an attempt to quantify this system he constructed a mathematical model using three variables of weather: temperature, pressure, and wind velocity [2], which were the basis for the three equations in his computer simulation. In 1961, when Lorenz set the initial conditions three decimal places shorter than needed, the computer gave an output far from its usual. This led to chaos theory, where a system is sensitive to its initial conditions and small errors prove to have enormous effects on larger chaotic systems as a whole.

Calculations

A set of ordinary differential equations (ODEs) were used to estimate the behaviour for each particle in the atmosphere and model the behaviour of the system. The Lorenz ODEs are represented by:

$$\frac{dx(t)}{dt} = \sigma(y - x) \tag{1}$$

$$\frac{dy(t)}{dt} = x(\rho - z) - y \tag{2}$$

$$\frac{dz(t)}{dt} = xy - \beta z \tag{3}$$

Variables x, y and z represent convective circulation, the horizontal temperature variation and vertical temperature variation [1]. To model the initial conditions: $x = 1$, $y = 1$ and $z = 1$ were used, and the original Lorenz constants were set with $\sigma = 10$, $\beta = 8/3$ and $\rho = 28$ [3], where σ represents the ratio of viscosity to thermal conductivity, β the width-to-height ratio, and ρ the temperature gradient in the system [1]. In this paper we investigate what occurs when one of the initial constants change and the effect that it has on the system. To do this our model used σ values of 5.06 and 0.506 to represent a likely scenario that could result from the misplacing of a decimal point. The equations were solved using the 'ode()' function from the 'deSolve' package in RStudio [4]. The model was simulated to run for 50 days and at a daily interval step size of 0.01. To explore the system's sensitivity to the changes in σ, simulations were run for both values of σ, and the resulting trajectories of x, y and z were recorded in the time series of x and phase space of x vs z.

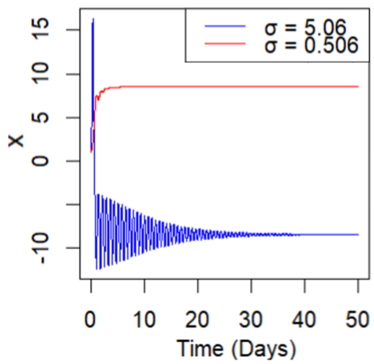

Figure 1: Graph depicting the particle behaviour in the x space over a period of 50 days.

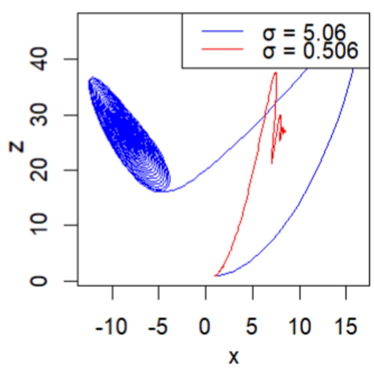

Figure 2: Graph depicting the phase space between x and z.

Discussion

When the value of σ is changed the characteristics of the graphs vary dramatically. For $\sigma = 5.06$, the graph in blue shows high oscillations and chaotic behaviour due to its continued oscillation in the 'x over time' graph and the formation of a looped structure also known as a 'Lorenz attractor' [3] shown in 'Phase space (x vs z)'. In contrast, for $\sigma = 0.506$ the system becomes stable quickly and the time series of x shows one oscillation before coming to rest at a positive value; this is reminiscent of being critically damped, and is in contrast to the underdamping seen in blue. The trajectory of the phase space is simpler, with values that do not repeat, which indicates that when σ is a lower value the system of the atmospheric particles becomes more stable. The results support the sensitivity of the Lorenz system to changes in initial conditions; as a result of a reduction in σ the model changes its characteristics which has led to an extremely different outcome.

Conclusion

The computational modelling of the Lorenz system highlights the sensitivity to changes in initial conditions. For larger values of σ, the behaviour of atmospheric particles become more chaotic, due to continuous oscillations and having a Lorenz attractor. As the value of σ increases the trajectory of the particles will create a 'butterfly-shaped' graph. However, when σ is reduced the path becomes more predictable, and as is evident from looking at both graphs, the behaviour plateaus. These results convey characteristics of chaos theory and its sensitivity to the parameters that are set.

References

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