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P4 2 Atmospheric Ignition

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Abstract

This paper examines the theoretical risk of triggering a chain reaction in Earth's atmosphere due to the detonation of a nuclear fission bomb during the Trinity test, part of the Manhattan Project. It explores nuclear fusion processes, specifically nitrogen fusion reactions, and calculates the energy and temperature thresholds required for such a reaction to occur. The analysis shows that the bomb's core reaches an upper limit temperature of approximately 1.01×10^{11} K, with an extremely low probability (1.27×10^{-19}) of initiating a chain reaction. This leads to the conclusion that the possibility of atmospheric ignition from a nuclear bomb detonation is virtually non-existent, despite concerns raised during the development of the bomb.

Introduction

This paper discusses the risk of a chain reaction in Earth's atmosphere caused by the detonation of a nuclear fission bomb, such as the one used in the Manhattan Project's Trinity test. This paper uses a different methodology as the safety factor of fusion gain and Bremsstrahlung is not considered due to the unknown cross-section factor of nitrogen.

Fusion Processes

To discuss the possibility of a chain reaction being set off, the nuclear reaction that would be involved in this process must first be understood. Among all the molecules present in the atmosphere, the nitrogen-nitrogen fusion reaction was considered 'the most dangerous assumption,' with additional consideration given to reactions involving protons [1].

$$^{14}\mathrm{N} + {}^{14}\mathrm{N} \longrightarrow {}^{24}\mathrm{Mg} + \alpha + 17.7\,\mathrm{MeV} \quad (1)$$

The probability of nuclear fusion from a fission bomb depends on the interplay between the Coulomb barrier and the Maxwell–Boltzmann distribution. The detonation generates high temperatures that increase particle energies, where most particles cluster around an average energy level. The Gamow window defines the specific energy range where fusion is most likely, showing that even under extreme conditions, fusion occurs mainly among particles within this narrow range [2]:

$$E_g = 2\mu c^2 (\pi \alpha Z_a Z_b)^2 \quad (2)$$

where, 'μ' is the reduced mass of the reactants, in this case, both nitrogen which is $\mu = 7$ a.m.u., 'α' being the fine-structure constant, the value of which is $\frac{1}{137}$ and 'Z'_a = 'Z_b' the atomic number of nitrogen.

The Gamow energy was calculated to be: $E_g = 16.46$ GeV.

The "Gadget"

The gadget refers to the atomic bomb being developed under the command of Dr. Robert

J. Oppenheimer. This section calculates the upper limit temperature of the core of the bomb. This is crucial as the temperature needs to be enough to overcome the Coulomb barrier to fuse the atom. It is known that the energy released on the detonation of the trinity gadget was equivalent to 21 kT of TNT [3] which is $8.78 \times 10^{13} J \approx 8.8 \times 10^{13}$ J (1 ton of TNT = 4.18×10^{9} J). The core of the bomb was made of 6.7 Kg of semi-solid ^{239}Pu sphere, the radius of which was 4.6 cm [4]. The instantaneous or upper limit temperature of the core can now be calculated using the specific heat capacity equation:

$$Q = mc_p \Delta T \tag{3}$$

where 'm' = mass of the core, 'c_p' = specific heat capacity of ^{239}Pu i.e. 130 J/Kg · K [5] , 'Q' is the energy released by the bomb, 'ΔT' is the upper limit temperature.

Using this equation, the upper limit temperature calculated is: $\Delta T = 1.01 \times 10^{11}$ K.

Gamow-Sommerfeld Factor

The Gamow-Sommerfeld factor is a probability factor for two nuclear particles' chance of overcoming the Coulomb barrier to undergo nuclear reaction. This probability is given by the following formula [2]:

$$P(E) = e^{-\sqrt{\frac{E_g}{E_t}}} \tag{4}$$

where, $E_t = K_b \Delta T$, the thermal energy [6]. This energy was calculated to be 8.7 MeV.

Solving equation 4, we get the probability as 1.27×10^{-19}, or in other words, zero.

Results and Discussion

The calculations performed in this study suggest that the probability of nitrogen-nitrogen fusion initiating a chain reaction in Earth's atmosphere due to the detonation of a nuclear bomb, such as the Trinity test, is extremely low. The Gamow energy for nitrogen fusion was calculated to be 16.46 GeV, a value far exceeding the thermal energy generated by the bomb. Using the specific heat capacity of ^{239}Pu, the upper limit temperature of the bomb's core was found to be approximately 1.01×10^{11} K, corresponding to a thermal energy of 8.7 MeV.

The probability of nitrogen nuclei overcoming the Coulomb barrier and undergoing fusion was calculated using the Gamow-Sommerfeld factor. The result, 1.27×10^{-19}, indicates that the chance of nitrogen fusion occurring in the bomb's environment is effectively zero.

Conclusion

In conclusion, the risk of setting the atmosphere of Earth on fire due to the detonation of an atomic bomb is practically zero. Although this is not a huge revelation nowadays since atomic bombs have been used previously, it was thought that it could be possible during the Manhattan Project.

References

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