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P2 8 Avoiding Exams To The Extreme

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Abstract

This paper determines the velocities and consequential forces needed to move the Earth to outside of the habitable zone (0.7 AU). A Hohmann transfer is used and requires two changes in velocity, first at 2,700 ms^{-1} to achieve an elliptical orbit and second at 2,900 ms^{-1} to achieve a closer circular orbit. The forces required are 1.87 x 10^{22} N and 1.34 x 10^{22} N respectively.

Introduction

A physics student wants to destroy the world, ideally before 20th May (exam period). One of their ideas is to move the Earth's orbit until it is outside of the habitable zone. The student wants to calculate the velocities and forces needed to achieve this.

Method

The definition of the habitable zone is the distance from a star at which liquid water could exist on an orbiting planet's surface [1]. For our Solar System, the habitable zone can be said to be between Venus (0.72 AU [2]) and Mars (1.52 AU [2]) [1]. Earth is assumed to be a particle, with a circular orbit at a distance of 1 AU from the Sun. As the Earth's current orbit is closer to the inner edge of the habitable zone, the student, desiring for the world to be destroyed quickly, has chosen to move the orbit closer to the Sun.

As a quick transfer time is needed, the Hohmann transfer is selected as a possibility as it is the most efficient two impulse method which connects two circular orbits that do not intersect [3]. This transfer consists of two changes in velocity, where the first, places Earth into an

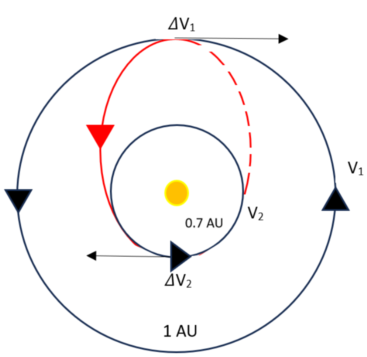

Figure 1: Hohmann transfer from a larger orbit to a smaller orbit.

elliptical transfer orbit and the second, adjusts the orbit into a circular orbit (to keep the Earth outside the habitable zone permanently). The time taken for the time between the two changes in velocities (Δv_1 and Δv_2 (red line Figure 1)) is found by applying Kepler's 3rd law, to be [4]:

$$t = \frac{\sqrt{\frac{4\pi^2}{GM}\left[\frac{r_1 + r_2}{2}\right]^3}}{2} \tag{1}$$

where M is the mass of the Sun (1.99 x 10^{30} kg), r_1 is the initial orbit of Earth (1 AU =

1.496 x 10^{11} m) and r_2 is the final orbit. To be completely outside of the habitable zone, r_2 can be presumed to be 0.7 AU (1.05 x 10^{11} m) and G is the gravitational constant 6.67 x 10^{-11} $\mathrm{Nm^2kg^{-2}}$. This would take approximately 143 days from Equation 1, which is within the desired time requirement. If the student had chosen to move Earth's orbit to the outer edge of the habitable zone (1.52 AU), then for a Hohmann transfer, it would take over 257 days, which is outside the time required.

Next is to determine the velocities needed to change from a circular orbit to an elliptical one. The first burn (change in velocity) is found by using Equation 2 [3]:

$$v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a}} \tag{2}$$

where μ is the standard gravitational parameter of the main body (GM) and r_1 is the initial radius of the Earth from the Sun 1.496 x 10^{11} m. a is the semi-major axis of the elliptical transfer orbit [3]

$$a = \frac{r_1 + r_2}{2} \tag{3}$$

where r_2 is the final radius, so 1.05 x 10^{11} m in this case. Therefore, the required change in velocity for the first burn is found to be 27,100 $\mathrm{ms^{-1}}$. Earth is already moving at 29,800 $\mathrm{ms^{-1}}$ from Equation 4 (r = 1.496 x 10^{11} m), so it will need to reduce by 2,700 $\mathrm{ms^{-1}}$.

$$v = \sqrt{\frac{GM}{r}} \tag{4}$$

Then, when the elliptical orbit reaches the desired orbit of 0.70 AU, a second burn will occur, where the required change in velocity can be found using Equation 5 [3]. A second burn is to create a new circular orbit, which keeps Earth out of the habitable zone.

$$v_2 = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a}} \tag{5}$$

The velocity needed for the second burn is 38,500 $\mathrm{ms^{-1}}$. At this orbit, a velocity of 35,600 $\mathrm{ms^{-1}}$ is needed for a circular orbit (from Equation 4), so a change of 2,900 $\mathrm{ms^{-1}}$ is required. If the student starts on 4th December, there are 167 days until the 20th May, however, the time between burns is 143 days, meaning there are 25 days left for the execution of the first burn to an elliptical orbit and then a second burn from an elliptical to a circular orbit. If 10 days are given for the first burn and 15 days to the second, the thrust needed for the Earth to decrease its velocity by 2,700 $\mathrm{ms^{-1}}$ is given by

$$F = m\frac{\Delta v}{\Delta t} \tag{6}$$

m is mass of the Earth (5.97 x 10^{24} kg), Δv is 2,700 $\mathrm{ms^{-1}}$ and Δt is 864,000 s. The force needed for the earth to change velocity in 10 days is found to be 1.87 x 10^{22} N. For the second burn, the thrust needed would be 1.34 x 10^{23} N as Δv is 2,900 $\mathrm{ms^{-1}}$ and Δt is 1,300,000 s.

Conclusion

The velocities needed for a two impulse transfer between two circular orbits (2,700 $\mathrm{ms^{-1}}$ and 2,900 $\mathrm{ms^{-1}}$) and the thrusts required; 1.87 x 10^{22} N for the first transfer and then 1.34 x 10^{22} N have been determined. Next, the student would need to find a viable method of moving the Earth for these two cases. However, at these velocities, the known world would have most likely been destroyed already, so exams might have been called off even before the Earth exits the habitable zone.

References

[1] De La Torre, B. 2022. *What Is the Habitable Zone?* NASA [Accessed 4 December 2023]

[2] NASA Jet Propulsion Laboratory. *Solar System Sizes and Distances* [Accessed 4 December 2023]

[3] Weber. B. *Hohmann Transfer* [Accessed 4 December 2023]

[4] `https://www.youtube.com/watch?v=O_EsXfVN988` [Accessed 4 December 2023]