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A3 6 Quakepocalypse

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Abstract

In this paper, we determined the magnitude that an earthquake would have in order to exceed the Earth's gravitational binding energy, destroying the planet, to be 18.83 on the Moment Magnitude Scale (the successor of the Richter Scale).

Introduction

Earthquakes are incredibly powerful and devastating natural phenomena that occur all over the globe. The strength of earthquakes was historically determined using the Richter Scale, but this has now been replaced by a more rigorous scale, which is called the Moment Magnitude Scale (M_W) [1]. This scale, put simply, is a measure of the shaking of the Earth's crust [2]. This scale is logarithmic, so an earthquake with $M_W = 8$ is 10 times more powerful than an earthquake with $M_W = 7$. The magnitude of an earthquake can be related to energy via the following equation [1]:

$$\log_{10}(E_{quake}) = 5.24 + 1.44M_W \tag{1}$$

, where E_{quake} is the energy released by an earthquake in Joules. This can be rearranged to:

$$E_{quake} = 10^{5.24+1.44M_W} \tag{2}$$

This means that for every integer increase in magnitude, the energy released by the earthquake increases by approximately 27.5 times. Using this, we can determine what earthquake magnitude would release enough energy to exceed Earth's gravitational binding energy, destroying the planet.

Gravitational Binding Energy of the Earth

The gravitational energy, E_{grav}, of an object is dependent on the object's mass, M and radius R. This is seen in the equation below [3]:

$$E_{grav} = \frac{3}{5} \times \frac{M^2G}{R} \tag{3}$$

, where G is the gravitational constant. For the Earth, $M = 5.97\times10^{24}$ kg and $R = 6.37\times10^6$ m. This leads to $E_{grav} = 2.24 \times 10^{32}$ J. In order to destroy Earth, an earthquake would have to release energy in excess of E_{grav}.

Quakepocalypse

The most powerful recorded earthquake occurred had a magnitude of 9.5 [4]. Using (Eq 2), this earthquake would have released 8.3×10^{18} J, which is considerably less than E_{grav}. As a result, we calculated the magnitude of an earthquake where $E_{grav} = E_{quake}$, so:

$$E_{quake} = 2.24 \times 10^{32} = 10^{5.24+1.44M_W} \tag{4}$$

Equation 4 can then be rearranged to find the magnitude M_W:

$$M_W = \frac{\log E_{quake} - 5.24}{1.44} = 18.83 \tag{5}$$

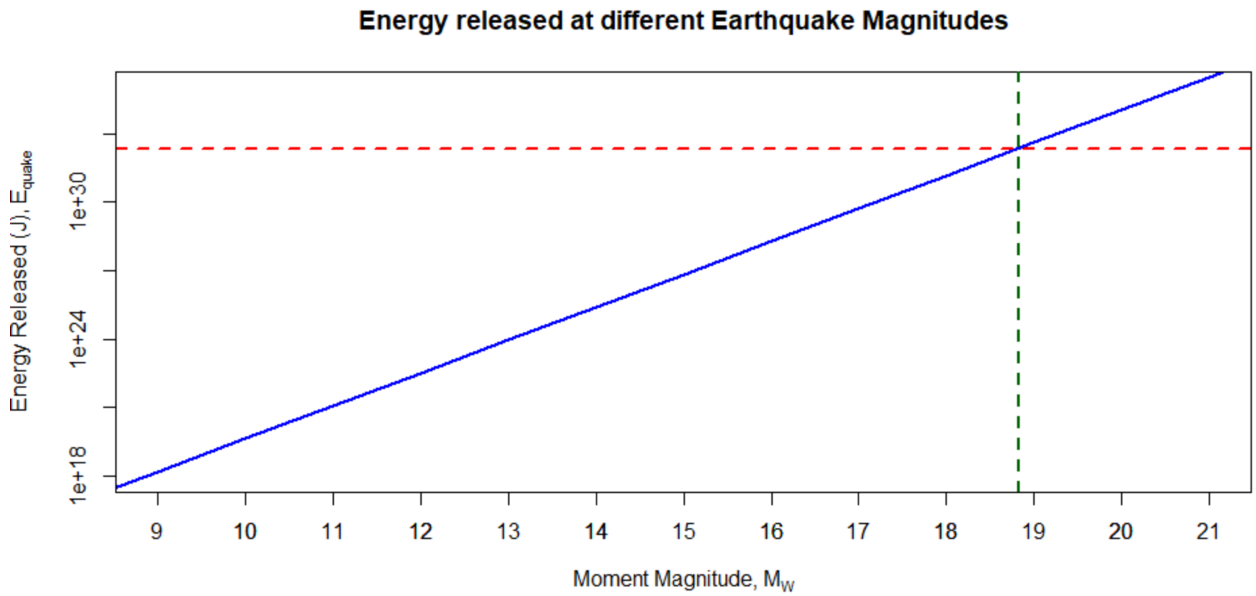

Figure 1: A logarithmic plot of earthquake energy vs earthquake magnitude. The red dashed line indicates E_{grav} and the dark green dashed line indicates the magnitude of an Earth destroying earthquake.

This means that an earthquake of magnitude 18.83 would have to occur in order for the Earth's gravitational binding energy to be overcome, breaking the Earth apart. In Figure 1, the relationship between the magnitude of an earthquake and the energy released is illustrated over a range of different magnitudes.

Discussion

The United States Geological Survey (USGS) states that an earthquake above a magnitude of 10 is impossible, as there is no tectonic fault line long enough to store the necessary potential energy before the fault line slips [4]. As seen in Figure 1, no feasible earthquake could occur to even come close to exceeding the Earth's gravitational binding energy. In fact, Figure 1 (alongside Equation 5) demonstrates that the maximum amount of energy that an earthquake can release cannot exceed 1×10^{19} J. Therefore, this 18.83 magnitude earthquake is, thankfully, impossible.

Conclusion

We have determined that earthquakes with a magnitude exceeding 18.83 would release enough energy to surpass the Earth's gravitational binding energy, destroying the planet. Fortunately, however, the USGS states that the structure of the Earth's crust forbids an earthquake occurring with a magnitude surpassing 10.

References

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