Journal of Physics Special Topics
An undergraduate physics journal

P3 7 Using Euler's method to find acceleration due to gravity on Skaro

O.E. Simpson, F. Astrophel, T. C. Miller, E. Rodgers
Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 15, 2023

Abstract
Using a scene from the BBC TV show Doctor Who, we have explored finding the acceleration due to gravity on an alien planet by the trajectory of a falling object. Using Euler's methods to take into account the drag, it seems that the value of g on Skaro is 2.2 ms^{-2}.

Introduction

In the 2015 episode of Doctor Who titled "The Witches Familiar", our protagonists find themselves on the alien planet Skaro. Within the episode one of our antagonists (Missy), endeavours to find the depth of a hole by pushing one of the characters (Clara) into it. After waiting to hear her crash she declares the holes depth as 20 feet.

Using her findings, the time of falling and accessory information about the falling object it is possible to estimate the acceleration due to gravity of this alien world.

Theory

We will be using Euler's method to accommodate for the expected drag of our body, with our drag force being expressed as:

$$F_D = \frac{\rho v^2 C_D A}{2} \tag{1}$$

Where F_D is the drag force, ρ is the density of the fluid, v is the speed of the object relative to the fluid, C_D is the drag coefficent and A is the cross sectional area of the the falling object.

The acceleration of our falling object (a), taking into account drag, can be modelled as:

$$a = \frac{W - F_D}{m} \tag{2}$$

Wherein m is the mass of our falling object as W is its weight, as always calculated as:

$$W = mg \tag{3}$$

With g being the acceleration due to gravity on the planet.

By plugging in both Eq. (1) and Eq. (3) into Eq.(2), we find that:

$$a = g - Kv^2 \text{ where } K = \frac{\rho C_D A}{2m} \tag{4}$$

As v is a changing variable of time, it seemed adept to use Euler's method to find our velocity at each point as:

$$v_{i+1} = v_i + (g - Kv_i^2)h \tag{5}$$

Where v_{i+1} is the current velocity, v_i is the previous velocity and h is the step (all other values are the same as above). Once the velocity has been calculated we can find the displacement of our object:

$$s_{i+1} = s_i + v_i h \tag{6}$$

Where s_{i+1} is the current displacement and s_i is the previous displacement.

Knowing the time it takes to fall the 20 feet described in the show, and with all else being constant but the velocity, by varying g until the fall time matches our expectations we can find the acceleration due to gravity on the surface of the planet.

Results

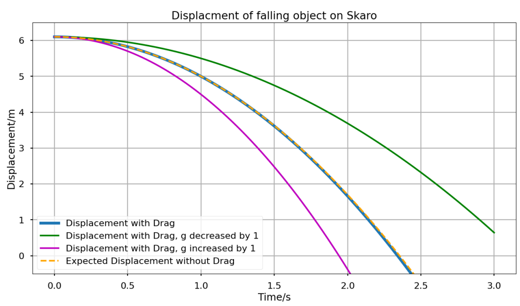

Figure 1: Graph showing the trajectory of the falling object on Skaro. The trajectory with this papers accepted value of g (2.2 ms^{-2}), taken with drag is shown in blue. The expected displacement without drag, with the same value of g is shown in dashed orange. For comparison the value of g increased by 1 is shown in magenta and the value decreased by one is shown in green.

Converting to metres our fall distance was 6.10 m. After repeat measurements the fall time appears to be 2.34 s.

As the human character is shown to breathe without issue, it will be assumed that the air density is the same as on Earth and ρ will be taken as 1.293 kgm^{-3}. The drag coefficient of a human (C_D) is estimated to be 1.0 [1]. Using information about the actress Jenna Coleman [2], it was found that she has mass of 50 kg and we estimated her surface area using her height and waist measurements to be around 0.45 m^2.

After experimenting with the models and values of g, taking h as 0.00001 over a total run of 3 s, with drag on object falling 20 ft in 2.34 s would be under an acceleration due to gravity of 2.2 ms^{-2}.

Conclusion

The value is certainly lower than we would expect as the movement of objects and people on the planet would not suggest such a low acceleration. While we may look for in universe explanations such as differences in air density or simply that 20 ft was either a rounded estimate or an outright lie (with the latter seeming more in character), it's more likely that 2.34 s was a matter of comedic timing as opposed to a way to show some scientific prowess.

Regardless, the estimate seems to hold as a value reflective of the data that we have at our disposal.

References

[1] The Engineering ToolBox (2004). Drag Coefficient. [online] Available at: https://www.engineeringtoolbox.com/drag-coefficient-d_627.html (Accessed: 07 November 2023)

[2] Celebportal (2022) Jenna Coleman Height weight body stats age family facts, Thecelebportal. Available at: https://thecelebportal.com/jenna-coleman-height-weight-body-stats-age-family-facts/ (Accessed: 07 November 2023).