Journal of Physics Special Topics
An undergraduate physics journal

A1 7 The Flying Umbrella

C. Tang, L. Armitage, M. Alexander

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 9, 2023

Abstract

The umbrella is a commonly used tool to avoid rain. However, in media, they are often used to slow descent. We consider a situation where a person is on top of a very tall tower, and as there is a fire, he has to jump off the tower. We calculate that the umbrella must have a diameter of at least 4 m to survive with a safe landing velocity of 5 m s^{-1}.

Introduction

Umbrellas are commonly used to keep the user dry in rain. In popular media, however, holding one often slows the user's falling speed. We consider a situation where a person is on top of a very tall tower, and there is a fire. To avoid the fire, he has to jump down. We are interested in finding out whether the person can use an umbrella as a substitute for a parachute to jump off the building safely. We also want to know how big the umbrella needs to be in order to survive with a safe landing velocity of 5 m s^{-1}, which is the landing speed for skydiving [1].

Method

We assume that the umbrella is strong enough so that it would not deform and can maintain its shape throughout the journey. Secondly, we assume that the umbrella is shaped like a hemisphere. Thirdly, we assume the umbrella has negligible mass. Also, the tower is very tall, so the person reaches terminal velocity during the descent.

To determine the terminal velocity of the person, we first have to find out the drag force F_{drag} (N) provided by the umbrella:

$$F_{drag} = \frac{1}{2} C_D \rho v^2 A \tag{1}$$

Here, C_D is the drag coefficient of the umbrella, v (m s^{-1}) is the velocity of the person, ρ (kg m^{-3}) is the density of air and A (m^2) is the surface area of the umbrella [2]. Since A is half of a sphere, $A = 2\pi r^2$, where r (m) is the radius of the umbrella.

Newton's second law states that the net force $F_{net} = ma$, where m (kg) is the mass of the person and a (m s^{-2}) is the net acceleration. At terminal velocity, $a = 0$, and the gravitational force F_g is balanced by the drag force, thus we have:

$$\frac{1}{2} C_D \rho v^2 A = mg \tag{2}$$

where g is acceleration due to gravity on Earth (= 9.81 m s^{-2}).

To determine the terminal velocity of the person, we rearrange Equation (2) for v and substitute in the expression for the area A, which results in:

$$v = \sqrt{\frac{mg}{C_D \rho \pi r^2}} \tag{3}$$

To find out the required area of the umbrella for the person to survive, we rearrange equation (2) for the radius r:

$$r = \sqrt{\frac{mg}{\pi C_D \rho v^2}} \tag{4}$$

Result

A typical large rain umbrella has a diameter of 1.70 m [3], so the radius of the umbrella r would be 0.85 m. The average mass of a person m is 60 kg [4], and the drag coefficient C_D is 1.42 for an empty hemisphere [5]. For air, $\rho = 1.23$ kg m^{-3} [2]. Using Equation (3), we estimate that the terminal velocity of the person would be 17.2 m s^{-1}. However, landing at this velocity is not safe.

Therefore, to survive unharmed, we need to know how big the umbrella has to be in order to reduce the velocity to 5 m s^{-1}. Using Equation (4) and substituting $v = 5$ m s^{-1} into the equation, using the same values of m, the radius of the umbrella required is calculated to be 2 m, which is over twice the width of the normal umbrella.

Discussion

For a human to survive the free fall with an umbrella unharmed, the diameter of the umbrella needs to be at least 4 m. A large commercial umbrella could only slow you down to 17.2 m s^{-1}, which would be dangerous for the person to land and would subject to potential injury. A higher falling velocity that a person survived on record was 35 m s^{-1} according to [6], the person was seriously injured [7].

In reality, finding an umbrella with a diameter of 4 m might not be possible, as this is even bigger than a beach umbrella. It would be difficult to hold this umbrella as well.

Although an umbrella has a hemispherical shape, which efficiently decelerates the person, an umbrella is different from a parachute in that the structure of the umbrella would deform easily as the person falls. On the other hand, a parachute can maintain its structure most of the time during the fall. Thus, having an umbrella that is strong enough to maintain its shape against air resistance would be difficult to achieve in the real world. Hence, a commercial umbrella cannot be used as a substitute for a parachute.

Conclusion

To conclude, a typical large-sized umbrella cannot help you to land safely from a very tall building. You would need a much bigger umbrella to survive. Therefore, you should never jump off tall buildings with only an umbrella in emergencies.

References

[1] Mr Riddz Science [online], Youtube (2016) `https://www.youtube.com/watch?v=EabUUrZFnFE` [Accessed 7 November 2023]

[2] `https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/rktvrecv.html` [Accessed 7 November 2023]

[3] `https://umbrellasandbeyond.com/products/gb-85168-gustbuster-doorman-umbrella#:~:text=GustBuster%2068%22%20Doorman%20Umbrella%20is,and%20chauffeurs%20throughout%20the%20world.` [Accessed 7 November 2023]

[4] S. C. Walpole, D. Prieto-Merino, P. Edwards, J. Cleland, G. Stevens and I. Roberts, BMC Public Health **12**, 4 (2012)

[5] `https://eaglepubs.erau.edu/introductiontoaerospaceflightvehicles/chapter/bluff-body-flows/` [Accessed 7 November 2023]

[6] `https://www.theguardian.com/science/2004/may/20/thisweekssciencequestions2` [Accessed 02 December 2023]

[7] `https://www.bbc.co.uk/news/world-europe-38427411` [Accessed 02 December 2023]