Journal of Physics Special Topics

An undergraduate physics journal

P5 3 Insane Tea Enthusiasts

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December 13, 2023

Abstract

Following our previous study, we continue the analysis of water boiling time in extreme pressure environments. We find the pressure required to extend the boiling time of water for three cups of tea to one hour. The result is 1.68×10^{10} Pa, which is comparable to pressures present in the upper mantle of Earth. The discussion of this finding leads to the conclusion that, under such circumstances, water transitions into its most dense state of ice called ice-VII.

Introduction

Our recent study was related to preparation of tea in the most extreme locations on Earth[1]. The goal was to get an understanding of how significant pressure differences affect our daily activities, and we found that the difference in the boiling time of water at the bottom of Mariana Trench and the summit of Mount Everest was approximately 7 minutes. These places were chosen in order to extrapolate the pressure dependence; however, in our opinion the difference was unexpectedly low. Hence, we decided to broaden our study by approaching the matter another way. Namely, by attempting to find what pressure could extend the time it takes to boil water for three cups of tea to one hour.

Method

We shall follow the reverse method of our previous paper, since now the water boiling time is given as 60 minutes and it is the pressure that is the unknown. Therefore, we begin with the time given from the definition of power as:

$$t = \frac{Q}{P} \tag{1}$$

where P is the power of a regular kettle (3,000 W[2]) and Q is the heat required for getting a certain amount of water to boil. The latter is calculated from the heat transfer formula:

$$Q = c_w m \Delta T \tag{2}$$

where c_w is the heat capacity of water (4,190 $\text{Jkg}^{-1}\text{K}^{-1}$), m is the mass of water needed to make three cups of tea (0.625 kg[1]) and ΔT is the temperature difference between the boiling point at pressure p, which is derived from the Clausius-Clapeyron equation[1], and the room temperature (293 K), given by:

$$\Delta T = (\frac{1}{T_0} - \frac{R}{H_w} \ln \frac{p}{p_{atm}})^{-1} - T_{room} \tag{3}$$

where T_0 is original boiling point (373 K), R is the gas constant (8.31 $\text{Jmol}^{-1}\text{K}^{-1}$), H_w is the molar enthalpy of water (40,700 Jmol^{-1}) and p_{atm} is the standard atmospheric pressure (101,000 Pa). Substituting both Equation 2 and 3 into Equation 1, gives the boiling time as:

$$t = \frac{c_w m}{P}((\frac{1}{T_0} - \frac{R}{H_w} \ln \frac{p}{p_{atm}})^{-1} - T_{room}) \tag{4}$$

Rearranging to find the pressure yields:

$$p = p_{atm} \exp\left(\frac{H_w}{R}\left(\frac{1}{T_0} - \left(\frac{Pt}{c_w m} + T_{room}\right)^{-1}\right)\right) \quad (5)$$

which, unfortunately, cannot be simplified into a more concise form and therefore, will be used as the final formula.

Results

The pressure required to extend the boiling time of water for three cups of tea to one hour, was determined to be 1.68×10^{10} Pa or alternatively 16.8 GPa. This is approximately two orders of magnitude higher than in the example of Mariana Trench[1]; which as a comparison corresponds to pressures present in the upper mantle of Earth[3]. This demonstrates how unrealistic experiencing such a water boiling time would be.

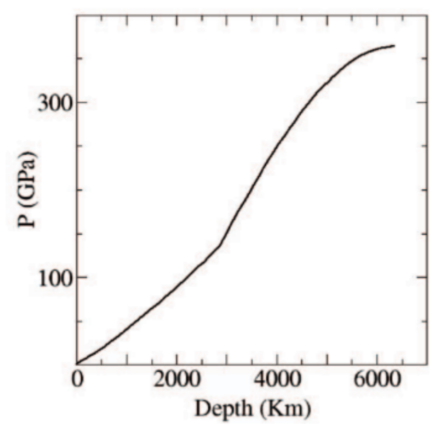

Figure 1: Pressure as a function of Earth's depth. The upper mantle ranges down to 410 km, which is where the pressure reaches values approximate to our estimated result[3].

Conclusions

We achieved our goal by finding the pressure causing the preparation time of three cups of tea to reach one hour of 1.68×10^{10} Pa. However, our analysis is purely hypothetical, since in reality water would undergo a phase transition into ice due to the high pressure forcing the rearrangement of water molecules into a more organised structure.

Disregarding the aspect of temperature, water would crystallise into *nota bene* the most dense form of ice called ice-VII under that pressure[4]. Thus, preparation of tea would also require supplying it with enough heat for transitioning to the liquid state, providing a different answer. Nevertheless, we believe our study of this topic is now fully concluded and that we successfully reached its goal.

References

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