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P3 5 Estimating Hubble's Constant with Redshift

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Abstract

Since the discovery of the expanding universe, there have been numerous attempts to determine Hubble's constant H_0. In this paper we attempt to estimate Hubble's constant using known relationships between expansion and redshift. We found $H_0 = 6.40 \times 10^{-18}$ Hz or 197 km s^{-1} Mpc^{-1}, which is almost 3x greater than the accepted value of $H_0 = 70$ km s^{-1} Mpc^{-1}. Reasons for this discrepancy, such as a simplistic model and inclusion of the cosmological constant, are outlined in the discussion.

Introduction

The discovery of the expanding universe is one the most profound revelations in physics. Evidence of expansion was presented by E. Hubble, who showed the galaxies' recessional velocity to be linearly proportional to its distance [1]. This parameter is now known as Hubble's constant H_0.

This paper details an estimate of Hubble's Constant using data from 15 galaxies with known redshift & distance. Dark matter and dark energy will not be considered.

Theory

The first Friedmann equation describes expansion of homogeneous isotropic space, and provides an expression by which H_0 can be estimated:

$$H_0^2 = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a(t)^2} \qquad (1)$$

Where $a(t)$ is the relative size or "scale factor" of the universe, ρ is the mass density of the universe, and k is a curvature constant. Wojtak & Prada show that the scale factor $a(t)$ and redshift z can be related using the below expression [2]:

$$a(t) = \frac{1}{1+z} \qquad (2)$$

Equations 1 & 2 are almost all that is needed in order to estimate Hubble's constant. The distance to an object in light years can be used as a substitute for time (with today being t = 4.35×10^{17} s or 13.8 billion years), as the light itself is not instantaneous; light rays from distant objects has spent linearly more time travelling towards us.

Method

The scale factor $a(t)$ for each object will be calculated from their redshift using equation 2. Using the light year distance as time, an expression relating scale factor and time will be made with polynomial regression using least squares fitting [3]. For simplicity, we will model $a(t)$ as a 3rd degree polynomial, which simplifies the calculation of the derivative $\dot{a}(t)$.

Substituting $\dot{a}(t)$ and $a(t)$ into equation 1, and substituting in $t = 4.35 \times 10^{17}$ s will allow us to find H_0. The galaxy data used was taken from The NASA/IPAC Extragalactic Database [4], and is in Table 1.

Result

Using the 3rd degree polynomial regression method, the expression for $a(t)$ to 2 s.f. was found to be:

$$a(t) = (9.0\times10^{-27})t^3-(2\times10^{-18})t^2+10^{-10}t \quad (3)$$

and its derivative was determined to be:

$$\dot{a}(t) = (2.7\times10^{-26})t^2-(4\times10^{-18})t+10^{-10} \quad (4)$$

Dividing equation 4 by equation 3 gives us an expression for Hubble's constant for any value of t. Substituting in $t = 4.35\times10^{17}$s returns a value for H_0, which was found to be $H_0 = 6.40\times10^{-18}$ Hz or 197 km s^{-1} Mpc^{-1} to 3 s.f.

Discussion

The true value for H_0 is still undetermined due to significant disagreements in estimations. This is likely due to some unknown factor affecting results from different methods, and is known as the *Hubble Tension* [5]. Despite this, a commonly used value for H_0 is around 2.27×10^{-18} Hz or 70 km s^{-1} Mpc^{-1}. This value is about 35% of the value estimated in this paper, but is still of the same order of magnitude.

The high result could be because the cosmological constant was not considered, so extra expansion due to dark energy was not factored out. Secondly, the models used may be too simplistic. While the model used meant only 2 data points for each galaxy are needed (reducing the total error), it is likely that more properties of the galaxies are needed in order to find a better estimate. For example, the redshift is severely influenced by the gravitational pull of nearby galaxies.

Conclusion

In this paper, we found an estimate for Hubble's constant H_0 to be 197 km s^{-1} Mpc^{-1}. This value is almost triple the accepted value for H_0 which is 70 km s^{-1} Mpc^{-1}. Reasons for the discrepancy such as a simplistic model and dark energy are discussed.

Appendix

Table 1: Objects and respective redshifts z, distances (Gly) & scale factor $a(t)$. All values are to 5 d.p.

Object	z	Distance (Gly)	$a(t)$
M87	0.00428	0.05349	0.50107
M51	0.00154	0.02316	0.50039
M49	0.00333	0.05590	0.50083
M58	0.00506	0.06230	0.50126
M60	0.00373	0.05669	0.50093
M61	0.00522	0.05251	0.50126
M64	0.00136	0.01729	0.50130
M66	0.00243	0.03131	0.50034
M83	0.00172	0.01520	0.50061
M89	0.00113	0.04999	0.50043
M94	0.00103	0.01601	0.50028
M99	0.00803	0.04524	0.50026
M101	0.00080	0.020874	0.50199
M104	0.00345	0.031148	0.50085
M109	0.00349	0.083496	0.50087

References

[1] Hubble, E., (1929). *A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae*, PNAS, Volume 15, Issue 3, pp. 168-173.

[2] Wojtak, R.; Prada, F., 2016, *Testing the mapping between redshift and cosmic scale factor*, MNRAS, Volume 458, Issue 3, Pages 3331–3340.

[3] McClave J. T.; Deitrich F. H., 2011, *Statistics (12th edition)*, Pearson, Chapter 12.1

[4] NASA/IPAC Extragalactic Database (NED) [Accessed 04/11/2023]

[5] Exploring The Hubble Tension [Accessed 05/11/2023]