

Journal of Physics Special Topics

An undergraduate physics journal

A5 5 Under the Sea

E. Matkin, L. Morriss and M. Shkullaku

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 5, 2023

Abstract

The paper looks at the pressure at different depths if the Earth was flooded with water up to the height of Mount Everest's peak. The pressure at the radius of the Earth would be 8.672×10^7 Pa and for the deepest point in the ocean (the Mariana Trench), it would be 1.937×10^8 Pa.

Introduction

In this paper, we calculate the volume and mass of water required to flood the Earth so that it reaches the highest altitude of land above sea level. Then, we calculate the pressure at the current sea level (at the average radius of the Earth) as well as the pressure at the deepest point in the ocean.

Flooding the Earth

We assume that the Earth is a perfect sphere with a radius of $R_E = 6378000$ m [1]. Then, we take the highest altitude above sea level (the peak of Mount Everest), which is $R_H = 8849$ m [2]. We then flood the Earth to the height of Mount Everest's summit to cover all the land. Any land above sea level is not taken into account as the average level of land above this is approximately 700 m and therefore does not have a significant effect on the results. First, we take the volume of a sphere given in Equation (1).

$$V = \frac{4}{3} \times \pi r^3 \quad (1)$$

Next, we found the volume of water (V_w) by taking the volume of a sphere at the height of Mount Everest's peak and subtracting the volume of the Earth. This is shown in Equation

(2).

$$V_w = \frac{4}{3}\pi(R_E + R_H)^3 - \frac{4}{3}\pi(R_E)^3 \quad (2)$$

Substituting in the values gave $V_w = 4.529 \times 10^{18}$ m³. We took the density of water to be $\rho = 999.1$ kg m⁻³ at 15°C [3] (the average surface temperature of the Earth[4]).

$$M = \rho V \quad (3)$$

Using Equation (3), the mass of the water was calculated as $M_w = 4.525 \times 10^{21}$ kg.

Atmospheric Pressure

To find the pressure at specific depths in the water, the atmospheric pressure is required. We assumed that the atmospheric composition and density remain unchanged from its present state.

$$P(z) = P_0 \exp\left(\frac{-z}{H}\right) \quad (4)$$

where z is the height above the Earth's radius, P_0 is the pressure at the radius of the Earth (1 atm = 101.3 kPa [6]) and H is the scale height which is given by

$$H = \frac{k_B T}{\langle m \rangle g} \quad (5)$$

where $\langle m \rangle$ is the average mass of a molecule in the atmosphere, T is the temperature, k_B is the Boltzmann constant and g is acceleration due to gravity. The composition of the Earth's atmosphere is approximately 21% O₂, 78% N₂ and 1% Ar [5]. There are other molecules but the contribution to the composition is so small that it has minimal effect and is therefore negated from this calculation.

To find the average mass of a particle, take the mass of each atom and for oxygen and nitrogen we multiply it by two (as two atoms are required to make up these specific molecules) and then multiply by the abundance percentage. Add the results for oxygen, nitrogen and argon together for the average mass of a molecule in the atmosphere. The mass of an oxygen, nitrogen and argon atom are 2.657×10^{-26} kg, 2.326×10^{-26} kg and 6.634×10^{-26} kg respectively [6]. The overall average mass of a particle in the Earth's atmosphere is given by $\langle m \rangle = (0.22 \times 2 \times 2.657 \times 10^{-26}) + (0.78 \times 2 \times 2.326 \times 10^{-26}) + (0.01 \times 6.634 \times 10^{-26}) = 4.811 \times 10^{-26}$ kg.

Use the result from $\langle m \rangle$ and Equation (5) to find the scale height of the atmosphere.

$$H = \frac{1.38 \times 10^{-23} \times 288.2}{9.806 \times 4.811 \times 10^{-26}} = 8429 \quad (6)$$

Therefore, $H = 8429$ m. Then, we used the scale height in Equation (4) to find the pressure of the atmosphere at the height of Mount Everest's peak. Hence, the pressure at this height is $P(z) = 37340$ Pa.

Underwater Pressure

To find the pressure at a certain depth Equation (7) is used.

$$P_w = P(z) + \rho g \Delta h \quad (7)$$

Where $P(z)$ is the pressure at the peak of Mount Everest and h is the depth of interest below the new sea level.

First, we looked at the pressure at the Earth's radius (the existing sea level) by using Equation

(7) and $\Delta h = 8848$ m. $P_{sl} = 37340 + 999.1 \times 9.806 \times 8849 = 8.672 \times 10^7$ Pa.

Then, we looked at the new pressure at the lowest point in the ocean, which is within the Mariana Trench (known as Challenger Deep) with a depth of 10920 m below the existing sea level [2]. $P_{MT} = 37340 + 999.1 \times 9.806 \times (8849 + 10920) = 1.937 \times 10^8$ Pa.

Conclusion

We found that the mass of the water required to flood the Earth to the height of the peak of Mount Everest is 4.525×10^{21} kg. The new pressure at the bottom of the Mariana Trench is 1.937×10^8 Pa (191200 atm). The new pressure at the radius of the Earth is 8.672×10^7 Pa, which is 856 atm. From the assumptions originally stated, the radius of the Earth is not constant as the radius towards the poles is smaller meaning a larger amount of water above this point would cause a higher pressure. Therefore, these results are not very reliable.

References

- [1] M. A. Khan, Equatorial radius of the Earth: A dynamical determination. *Bull. Geodesique* **109**, 227–235 (1973).
- [2] <https://www.tranquilkilimanjaro.com/mariana-trench-deepest-point-on-earth-vs-mount-everest-highest-point-on-earth/> [Accessed 19 October 2023]
- [3] <https://www.usgs.gov/special-topics/water-science-school/science/water-density#overview> [Accessed 21 October 2023]
- [4] <https://science.nasa.gov/resource/solar-system-temperatures/> [Accessed 21 October 2023]
- [5] <https://www.noaa.gov/jetstream/atmosphere> [Accessed 21 October 2023]
- [6] P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers* 7 (W. H. Freeman, New York, 2008), Vol. 6, p. AP-3 - AP-7.