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A5 8 In Hot Water

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Abstract
This paper compares different methods for producing enough energy for every household in the United Kingdom to boil a kettle at the same time. We found that the amount of CO_2 that would be produced from burning coal to achieve this would be 2.32×10^6 kg. Another method would be to provide 1.71×10^8 solar panels.

Introduction

This paper looks different ways of powering kettles to boil water for every household in the UK. It describes the amount of coal required as well as the CO_2 produced from burning this. It also calculates the number of solar panels required to produce the same amount of energy.

Energy Required

The number of households in the UK in 2021 was approximately 28.1 million [1]. Therefore, if every household had one kettle each, that would mean that there would be 2.81×10^7 kettles. A standard mug has a volume of around 250 ml so boiling enough water for 4 cups would mean each kettle would need 1 litre of water. An average kettle has the power consumption of 3000 W [2]. Assuming that all of the energy goes to heating the water and is not lost to the surroundings, the total energy required to heat the water to 95 °C from room temperature (20 °C) can be found by using the formula for specific heat capacity.

$$q = cm\Delta T \tag{1}$$

Where q is the energy required, c is the specific heat capacity, m is the mass of the water and ΔT is the change in temperature. For this case, c = 4190 J kg^{-1} C^{-1} [3], m = 1 kg (as for water 1 l $\approx$ 1 kg) and $\Delta T = 95.0 - 20.0 = 75.0$ °C. Substituting these values into Equation (1) gives a value for the energy required for one kettle, q = 314000 J. The overall energy required to boil a kettle in every household in the UK is 8.82×10^{12} J. As the power of the kettle is 3000 J s^{-1}, it would take 105 s (just under 2 minutes) for the kettle to boil the water.

Burning Coal

The percentage of useful energy from burning coal to produce electricity is approximately 40% [4]. One ton of coal produces 2.21×10^{10} J, therefore 8.86×10^9 J (40% of the total energy) gets converted into electricity. This means that 995 tons of coal is required to power all of the household kettles (assuming no energy is lost through the wires). As 1 ton $\approx$ 907 kg, this is approximately 903000 kg of coal.

On average, for coal to burn it has to have a composition of 70% carbon [5] (which is 632000 kg for the overall coal mass). It is assumed that the coal burns entirely and each carbon atom becomes carbon dioxide. The mass of a carbon atom is 12.0107 u = 1.99×10^{-26} kg [3]. The overall amount of carbon atoms in the coal is 3.18×10^{31}. As two oxygen atoms are required to react with carbon to produce CO_2, the number of oxygen atoms are 6.36×10^{31}.

The mass of an oxygen atom is 15.9994 u = 2.66×10^{-26} kg [3]. Therefore, the total mass of the oxygen atoms are 1690000 kg. So for 903000 kg of coal burnt, 2.32×10^{6} kg of CO_2 would be produced.

Solar Panels

Solar panels absorb photons from the Sun into cells that then produce a current. Therefore, to find how much electricity can be produced by the solar panels, the amount of solar radiation incident on the solar panels is required. First, the luminosity of Sun is found.

$$L = \sigma T^4 4\pi r_s^2 \tag{2}$$

Where L is the Sun's luminosity, σ is the Stefan-Boltzmann constant, r_s is the Sun's radius and T is the Sun's temperature.

To find the incident radiation flux (F) at the Earth's surface use Equation (3).

$$F = \frac{L}{4\pi R^2} \tag{3}$$

Combining Equations (2) and (3) gives

$$F = \sigma T^4 \frac{r_s^2}{R^2} \tag{4}$$

By substituting in the values T = 5780 K, σ = 5.67×10^{-8} W m^{-2} K^{-4}, $r_s = 6.96\times10^{8}$ m and $R = 1.50\times10^{11}$ m (distance from the Sun to the Earth) [3], F = 1360 W m^{-2} (solar constant).

Assume that the solar panels face the Sun directly unobstructed and that the solar radiance does not get absorbed by the atmosphere. Assume the solar panels have an efficiency of 18% and the average size is 1 m by 2 m [6]. So, the average solar radiation input is = 1360 × 1 × 2 = 2720 W, but the efficiency means only 490 W is useful and becomes electricity. Therefore, for 8.82×10^{12} J over 105 s would require 1.71×10^{8} solar panels. However, in reality, this would be higher as the solar panels are not on the equator, the solar panels are likely obstructed either by shade or something else such as dust, and the atmosphere absorbs some of the Sun's radiance meaning the incident radiation is lower.

Conclusion

Overall, to boil one kettle in every household in the UK (28.1 million), it would require 903000 kg of coal to be burnt, which would produce 2.32×10^{6} kg of CO_2. Alternatively, 1.71×10^{8} solar panels would be required. This corresponds to an area approximately 3.42×10^{8} m^2 which is around 0.14% of the United Kingdom's surface area [7].

References

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