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P3 2 Walking on A Flat Earth

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Abstract

Despite the Earth being a globe, a small population of people believe the Earth is a flat disc. The inconsistent gravity would mean humans would need to walk at angles to stay balanced. We derive an expression for this tilt and find the angle to be $\theta = \arccos\left(\frac{A}{r}\right)^2$. We discuss not only how this disproves flat Earth but also one absurd solution flat Earthers provide to solve this dilemma.

Introduction

It has been known since Eratosthenes that the Earth is spherical [1], one simple proof being that there is a near-uniform inwards gravitational force on the Earth's surface. This would not be the case on a flat disc Earth.

Instead, an observer on a flat Earth would experience a different force depending on distance from the disc's centre. Thus in order to maintain balance, one must stand and walk at an angle. In this paper, we will derive an expression for this tilt.

We will assume most of the laws of physics are unchanged and will be using Newton's Laws of Motion [2].

Derivation

To begin, we must construct an alternative version of Newton's Law of Universal Gravitation, in order to describe a disc instead of a sphere.

Consider a flat disc Earth of radius R, within it is an annulus of radius r, and an infinitesimally small thickness dr (figure 1).

The flat Earth itself will have some depth δ, and the mass density will be labelled ρ. The mass element dm of this annulus will simply be the mass density multiplied by the area of the annulus and disc depth as below:

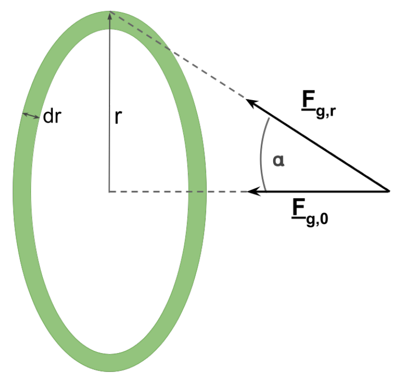

Figure 1: A simple diagram of an annulus of radius r and thickness dr.

$$dm = \rho\, \delta\, 2\pi r\, dr \quad (1)$$

Integrating equation 1 over the surface of the disc with respect to the radius will provide us with an expression for the enclosed mass M_{enc} of the annulus.

$$M_{enc} = \int_0^R dm\, dr = \rho \delta \pi R^2 \quad (2)$$

We then use Newton's Second Law of Motion to create an expression for the gravitational force, $F_{g,r}$:

$$F_{g,r} = M_{enc}\, g \tag{3}$$

$$F_{g,r} = g\, \delta\, \rho\, \pi r^2 \tag{4}$$

Finally, we may construct an expression for the angle, θ, at which the gravitational pull and the normal surface force are balanced. This "balancing" angle is equivalent to the angle subtended from the centre of the disk (at $r = 0$ and the observer at the edge of the annulus (see figure 1).

Hence, the angle can be determined using simple trigonometry:

$$\cos(\theta) = \frac{F_{g,0}}{F_{g,r}} \tag{5}$$

Ideally, equation 4 is then substituted into equation 5 and then simplified, however this becomes an issue because $F_{g,0}$ is 0. At the centre of the disc all gravitational forces balance each other, and will reduce our angle to be constant which is *physically incorrect*.

One option to rectify this issue is to set $F_{g,0}$ to be a constant, A. This would allow the angle to change with distance and keep the final expression intact. This could be physically viable since there is some gravitational pull due to the depth δ directly underneath an observer at the centre. Hence, A could be considered as a "calibration constant" to be measured similar to the gravitational constant G.

Substituting equation 4 into equation 5 but including A, and then simplifying gives us:

$$\cos(\theta) = \frac{g\, \rho\, \delta\, \pi r_0^2}{g\, \rho\, \delta\, \pi r^2} = \frac{A^2}{r^2} \tag{6}$$

Final Expression & Discussion

Rearranging equation 6 gives us our final formula for the angle:

$$\theta = \arccos\left(\frac{A}{r}\right)^2 \tag{7}$$

Which describes the balancing angle at any distance r from the centre of a flat Earth.

While the expression derived appears somewhat sound, it also demonstrates the absurdity of the flat Earth theory as there is no documented evidence for this tilt. To bypass this dilemma, some flat Earthers propose a Theory of *Universal Gravitation*, where gravity does not exist but instead the Earth and universe is accelerating "upwards" at g ms^{-2} [4]. This, however, introduces the issue of the Earth reaching velocities above the speed of light within a year of accelerating.

As of now, there is **no** flat Earth theory that can explain the disc shape and simultaneously the lack of tilt observed.

Conclusion

In this paper, we explored the physics surrounding flat Earth and one consequence of living on a disc: the need for a balancing tilt that increases with distance to the centre. We used simple laws of motion to derive an expression for this tilt and discussed the absurdity of the result and flat Earth in general.

References

[1] Cleomedes, *Caelestia*, pages 49–52.

[2] Newton, I; Chittenden, N. W.; Motte, A; Hill, T. P (1846). *Philosophiae naturalis principia*, page 74, definition IV, University of California Libraries.

[3] Newton, I; Chittenden, N. W.; Motte, A; Hill, T. P (1846). *Philosophiae naturalis principia*, pages 221-223 theorems 35 and 36, University of California Libraries.

[4] Tausami, Bishop, T.; 2013, *Universal Acceleration*, The Flat Earth Society Organisation.