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P1 4 Beau and Arrow

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Abstract

This paper explores the possibility of firing an arrow along the horizontal plane to determine if it would hit the bull's eye - splitting a pre-existing arrow embedded in the target. The initial velocity of the arrow was calculated to be 125 ms^{-1}. It lies within the expected range of velocities for recurve and longbow archers. Considering Newtonian projectile motion and the effects of air resistance, the arrow would strike at a distance of 0.368 m below the bull's eye. As a result, the arrow would not split the stationary arrow as depicted in the film *Brave*.

Introduction

In the classic 2012 Disney animation *Brave*, we follow the protagonist, Merida, on a journey through her teenage years. During an archery contest held by her parents, three potential suitors attempt to score a bull's eye with a bow and arrow. Merida, a keen archer, decides to take matters into her own hands, firing three arrows at the same targets. Her third shot perfectly hits the pre-existing arrow, splitting it. Considering the effects of air resistance, this paper analyses the projectile motion exhibited by her third arrow and whether this performance is achievable in real life.

Method and Results

Being a barebow archer, we assumed Merida fired at a distance of $x = 50$ m [1] along the horizontal plane. Timing the scene's duration, the arrow takes 0.4 s to complete its journey from release, to hitting the target. Using the speed-distance-time relation, we found the initial velocity of the arrow to be 125 ms^{-1}. This lies within the expected velocities of recurve bow arrows and longbow arrows [2]. In the animation, Merida fires all three arrows at a zero inclination angle to the horizontal. By considering the effects of air resistance, we have determined that the arrow will not hit the bull's eye. The calculations below demonstrate that the arrow will miss the target by a vertical displacement of -0.368 m. The set-up of the scene is illustrated in Figure 1.

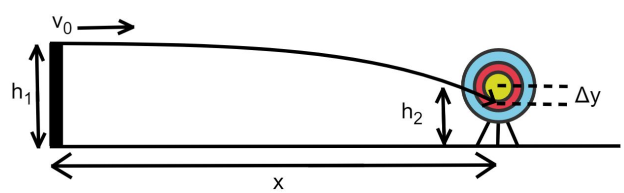

Figure 1: The path of the projectile motion followed by Merida's arrow after release from the barebow to strike the target.

From Figure 1, h_1 is Merida's height 1.63 m [3], h_2 is the distance from the ground to the bull's eye 1.22 m [4], v_0 is the initial velocity 125 ms^{-1}, and Δy is the distance by which the arrow misses the bull's eye 0.368 m. Using Newton's second law, we equate the net force to the weight and the velocity vector, arriving at Equation 1.

$$m\frac{d\mathbf{v}}{dt} = m\mathbf{g} - c\mathbf{v} \tag{1}$$

m is the mass of the arrow, 10.6 g [5], $\mathbf{v}$ is the velocity vector, $\mathbf{g}$ is the acceleration due to gravity, 9.81 ms^{-2}, and c is a positive constant for the later integration.

To determine the offset of the arrow caused by air resistance, we considered the motion in the y direction only. The vertical acceleration is given by

$$\frac{dv_y}{dt} = -g(1 + \frac{v_y}{v_t}) \tag{2}$$

where v_t is the terminal velocity of the arrow and v_y is the velocity in the vertical direction.

$$v_t = \frac{mg}{c} = (\frac{2mg}{\rho A C_d})^{1/2} \tag{3}$$

With radius 4.80 mm [5] the area A can be calculated. The density of air ρ, 1.29 kgm^{-3} [6], and the drag coefficient of the arrowhead is approximated to C_d, 0.1 [7], giving a terminal velocity value of $v_t = 150$ ms^{-1}.

Integrating Equation 2 gives the vertical velocity of the arrow.

$$\int_{v_0 sin(\theta)}^{v_y} -g(1+\frac{v_y}{v_t})dt = v_0 sin(\theta)e^{\frac{gt}{v_t}} - v_t(1-e^{\frac{gt}{v_t}}) \tag{4}$$

θ is the inclination angle to the horizontal. Integrating Equation 4 gives the vertical displacement.

$$y = \frac{v_t}{g}(v_0 sin(\theta) + v_t)(1 - e^{\frac{-gt}{v_t}}) - v_t t \tag{5}$$

We found the vertical displacement due to drag forces was -0.778 m. From Figure 1, the height difference between Merida and the target is 0.410 m. From these values, the distance the arrow misses the bull's eye is 0.368 m.

Conclusion

Considering the effects of air resistance, we have shown that Merida's arrow would not strike the target's bull's eye. Consequently, Merida's arrow would not split the pre-existing arrow. The effects of air resistance would cause the arrow to miss by a vertical displacement of -0.368 m. Merida doesn't consider simple Newtonian physics as she should have fired the arrow above the horizontal, accounting for drag, to split the arrow successfully.

References

[1] https://archerygb.org/(Accessed 22 October 2023)

[2] https://www.soarvalleyarchers.com/(Accessed 22 October 2023)

[3] https://www.groupeproxim.ca/en/article/health/average-height-and-weight(Accessed 22 October 2023)

[4] https://www.dimensions.com/collection/archery(Accessed 22 October 2023)

[5] https://grexluporum.blogspot.com/(Accessed 5 November 2023)

[6] https://www.earthdata.nasa.gov/(Accessed 5 November 2023)

[7] https://fazpublishing.com/jaat/index.php/jaat/article/view/9/9(Accessed 5 November 2023)