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A1 3 The Moving Earth

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Abstract

Supernova explosions happen all over the Universe. We consider an unlikely situation where our Sun gains mass and undergoes a supernova explosion. We estimate that if the Sun explodes at 8 solar masses, the momentum of the explosion causes the Earth to move away at a velocity of 14 km s^{-1}.

Introduction

Supernova explosions happen across the Universe. These explosions are often destructive, where stars are heated up and blow off planets from their orbits. The Sun cannot explode in a supernova as it is not heavy enough. However, it is still possible for the Sun to gain mass by accretion of matter in the Universe. For example, the Sun can gain mass if asteroids and comets hit its surface. We are interested in finding what would happen to the Earth if the Sun gains mass and a supernova occurs.

Method

The minimum stellar mass required for a supernova (and the new mass of the Sun) is around 8 solar masses [1], where 1 solar mass is 2×10^{30} kg [2]. In a supernova explosion, 75% of the mass of our Sun ($M_{ejected} = 1.2 \times 10^{31}$ kg) will be ejected at a velocity of 15 000 km s^{-1} uniformly in all radial directions [3] [4]. We assume the Earth is not blocked by other planets and the moon during the explosion.

Equations

We assume that the momentum of the Earth-Sun system is conserved. Thus, the momentum of the ejected mass impacting Earth must be equal to the momentum gained by Earth [2], shown in Equation (1).

$$M_{impacted} v_{ejected} = M_{Earth} v_{Earth} \tag{1}$$

Here, $M_{impacted}$ (kg) is the total mass hitting Earth, $v_{ejected}$ (m s^{-1}) is the velocity of the mass ejected from the Sun, M_{Earth} (kg) is the mass of Earth and v_{Earth} (m s^{-1}) is the velocity of the Earth after the collision.

We assume the ejected mass will be distributed uniformly in a sphere with a surface area equal to $4\pi R^2$, where R (m) is the radius of the sphere, i.e. the distance from the centre of the Sun. This means when the mass collides with Earth, R will be the distance from the Sun to the Earth, 1.5×10^8 km. The fraction of the ejected mass that will collide with Earth depends on the ratio of the cross-sectional area of Earth to the surface area of the ejecting sphere, shown in Equation (2):

$$M_{impacted} = \frac{M_{ejected} \pi R_{Earth}^2}{4\pi R^2} = \frac{M_{ejected} R_{Earth}^2}{4R^2} \tag{2}$$

By substituting Equation (2) into Equation (1) and rearranging for the velocity of Earth, we obtain Equation (3):

$$v_{Earth} = \frac{M_{ejected} R_{Earth}^2 v_{ejected}}{4 M_{Earth} R^2} \tag{3}$$

Result

Substituting $M_{Earth} = 6 \times 10^{24}$ kg, $M_{ejected} = 12 \times 10^{30}$ kg, $v_{ejected} = 15000$ km s^{-1}, $R_{Earth} = 6400$ km, and $R = 1.5 \times 10^8$ km into Equation (3) gives Earth a velocity of 14 km s^{-1} [2].

Discussion

For Earth to move with an extra velocity of 14 km s^{-1}, it needs to gain 6×10^{32} J of kinetic energy, which is a huge amount. Depending on the mass left in the core of the Sun, Earth may be able to escape the Sun with this velocity.

However, if this huge amount of kinetic energy is transferred to Earth, it is possible that Earth would deform. Consider the gravitational binding energy of Earth U (J), which is calculated using equation (4) from [5]:

$$U = \frac{3 G M_{Earth}^2}{5 R_{Earth}} \tag{4}$$

Here, G is the gravitational constant (=6.67×10^{-11} N m^2 kg^{-2}) [2].

Substitute in $M_{Earth} = 6 \times 10^{24}$ kg and $R_{Earth} = 6400$ km into equation (4), the binding energy of Earth is 2×10^{32} J. The binding energy of Earth is smaller than the gain in kinetic energy, which is 6×10^{32} J. Therefore, it is possible that the Earth can deform.

However, Earth is less likely to deform in reality because other planets and the moon can be in front of Earth so that they would can block part of the explosion. Also, not all of the impacting energy would be converted to Earth's kinetic energy, as some of the energy would be converted to heat instead. Therefore, the amount of energy received on Earth would be smaller than we calculated, but it would not be ideal for humans to live on Earth even if Earth does not deform. This is because the intense radiation released by the Supernova explosion would penetrate and decompose the Earth's atmosphere [6].

Conclusion

In conclusion, if the Sun gains mass to a total of 8 solar masses and undergoes a supernova explosion, the momentum would move the Earth at a velocity of 14 km s^{-1} away from the centre of the explosion. Earth moving with this velocity would not be ideal, and the Earth would also become inhabitable. It would be difficult for humans to live on Earth as plants might not survive without sufficient sunlight. This would be a challenging environment for humans to survive on Earth.

References

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