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P1 2 Green Shell Launch Velocity In Mario Kart

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Abstract

This paper investigates the required velocity for a green Koopa Troopa shell to cause various characters' karts to "spin-out" as shown in Mario Kart gameplay. By modelling the three karts as squares of different sizes and using their moment of inertia combined with the conservation of kinetic energy, the shell velocity could be calculated. The velocity required to cause two rotations per second for Toad, Yoshi, and Bowser was found to be 24.2 ms^{-1}, 57.2 ms^{-1}, and 221 ms^{-1} respectively.

Introduction

Green Koopa Troopa shells have been a recurring item in Mario Kart, being used in most games since their first appearance in Super Mario Bros[1]. Their main function is to be thrown directly at other karts, to cause a "spin-out" or to bounce off of the walls until they cause a collision or expire.

Here, we assume that a green shell (thrown by a player from behind) directly impacts the back left wheel of a square kart in front, at an angle perpendicular to the line between the target wheel and the centre of mass of the kart. In the game, green shells break upon impact and so we assume all of its kinetic energy will be transferred to the vehicle. This paper determines the velocity upon impact required for a green shell to cause a selection of characters' karts to make two full rotations in one second (whilst ignoring friction and air resistance).

Method

First, the mass of Toad, Yoshi, and Bowser are 6.35 kg, 60 kg, and 725 kg respectively[2]. Comparatively, given their weight class in the game being light, medium, and large characters, each of their karts could be assumed to relate in the same way. This meant that Toad's kart was estimated as 68 kg[3], the mass of Yoshi's kart was approximated to be 125 kg[3], and Bowser's was found to be 272 kg[3].

Next, the go karts were assumed to be squares each with a side length of 1 m, 1.5 m, and 2.5 m to account for their varying masses. In addition, the mass of Koopa Troopa was estimated to be 10 kg. However, we have assumed that Koopa Troopa shells are empty and break upon impact. Therefore, to determine the mass of the shell alone we can consider Koopa Troopa to be analogous to a tortoise, where tortoise species have a shell mass to body mass ratio within the range of 33.5 % to 52.3 %[4]. By using 33.5 %, since Koopa Troopa has a considerably larger head and limbs compared to a regular tortoise, the mass of a green shell was found to be 3.35 kg.

To determine the velocity required for a green shell to cause a kart to "spin out", the linear ki-

netic energy of the shell can be equated to the rotational kinetic energy of the target kart. Where m_s is the mass of the shell, v is the required velocity, I is the moment of inertia of a square, and w is the angular velocity.

$$KE = \frac{1}{2}m_s v^2 = \frac{1}{2}I\omega^2 \tag{1}$$

From "Mario Kart: Super Circuit", it is shown in the game play at 2:05 that after a collision with a green shell, the kart rotates twice in one second[5]. Therefore the angular velocity, ω, is 4π rads^{-1}. Next, the moment of inertia of a square can be derived starting from the standard moment of inertia equation:

$$I = \int r^2 dm \quad where \quad r^2 = x^2 + y^2 \tag{2}$$

We can start by defining the mass unit, dm, in terms of the density, ρ, and the area, A, where m_T is the combined mass of the character and their kart, as follows.

$$\rho = \frac{m_T}{A} = \frac{m_T}{xy} \quad and \quad dm_T = \rho dA = \rho dx dy \tag{3}$$

By applying this substitution of dm we can form a double integral over the x- and y-axes, where L is the side length of the kart, as shown below.

$$I = \rho \int_{\frac{-L}{2}}^{\frac{L}{2}} \int_{\frac{-L}{2}}^{\frac{L}{2}} (x^2 + y^2) dx dy = \rho \frac{L^4}{6} = \frac{m_T L^2}{6} \tag{4}$$

This result for the moment of inertia, and the angular velocity, ω, of 4π rads^{-1} can be substituted into Equation 1 and rearranged for the velocity of the shell, v to give Equation 5.

$$v = 4\pi L \sqrt{\frac{m_T}{6m_s}} \tag{5}$$

By substituting in the green shell mass, 3.35 kg, and each value of m_T and L, with 74.35 kg and 1.0 m for Toad, 185 kg and 1.5 m for Yoshi and 997 kg and 2.5 m for Bowser, we can find a velocity required for each to spin-out. This was calculated to be 24.2 ms^{-1}, 57.2 ms^{-1}, and 221 ms^{-1} respectively. This relationship is shown in Figure 1.

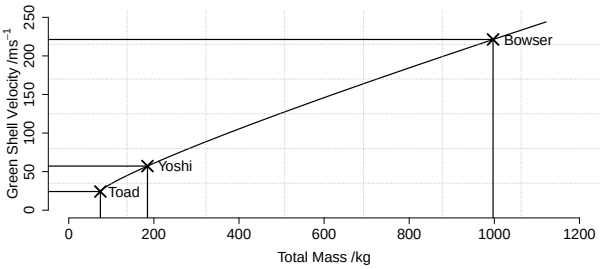

Figure 1: Model of the relationship between green shell velocity and the mass of the kart and the player combined.

Conclusion

Our calculations show that the required velocity upon impact that a green shell in Mario Kart would need to travel to cause a spin-out of two full rotations per second was 24.2 ms^{-1} (54.1 mph), 57.2 ms^{-1} (128 mph), and 221 ms^{-1} (495 mph) for Toad, Yoshi, and Bowser's karts. These values seem impressive but it should be noted that since we have ignored friction and air resistance, the velocities calculated reflect a situation where the karts would spin out forever. In addition to this, the velocity values calculated are too low since there would be friction and air resistance in real life.

References

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