Journal of Physics Special Topics
An undergraduate physics journal

A2 2 Damped Bungee Jumping

S. Cooper, E. Steele, C. Hamade

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 14, 2023

Abstract

In this paper we model a fast, quiet and safe rappelling method for the special forces. To do this we found a damping coefficient c and spring constant k that fulfilled our desired criteria. We found these to be $c = 99.09$ Ns/m and $k = 24.55$ N/m. By comparing to a standard value we found that our result was reliable and the final kinetic energy of a soldier to be negligible.

Introduction

In the special forces, silence during a tactical mission can be crucial. We consider a situation involving a soldier repelling down a building. While jumping is the quickest way down to the ground, it is very dangerous. Therefore we model a system of a critically damped bungee cord as the repelling device, this would be the fastest method while maintaining the element of surprise. To do this, we find suitable values of the spring constant k and the damping coefficient c that result in the desired system.

Methodology

To model the fall with a bungee cord, we start with a simple harmonic motion given by the second order differential:

$$mx'' - cx' - kx = 0 \tag{1}$$

Where the first, second and third terms are forces due to gravity with mass m (kg), damping force with coefficient c (Ns/m), and the resistance force due to the cord tension with spring constant k (N/m). The system starts from a maximum height A and is described by the general solution:

$$x = Acos(\sqrt{k/m}t) \tag{2}$$

To critically damp this system and find k, we started by determining the damping coefficient using the second term of equation (1).

$$F_{Damp} = -cv_0 \tag{3}$$

Where in this case the damping force is equal to the tension in the cord, so therefore the weight of the soldier, and v_0 is the initial velocity of the soldier when damping comes into effect. We decided that it was reasonable to use the basic motion equations to find the initial velocity. We found the time it took for the soldier to free fall the length of the unstretched bungee, and then used this to find v_0. Finally, we used the critical damping regime to calculate our k value:

$$c^2 = 4mk \tag{4}$$

and found the critical damping solution to be an exponential in the form of [1]:

$$x = e^{\gamma t}[A + (v_0 + \gamma A)t] \tag{5}$$

Where γ is:

$$\gamma = c/2m \tag{6}$$

We used the displacement equation (5) to model the system and determine the time of impact. We then differentiated equation (5) to find a function for the velocity and thus the final velocity. Finally we equated this velocity to the kinetic energy of the soldier to gauge how safe and silent the fall is.

Initial Conditions and Assumptions

Our initial conditions are as follows: Bungee cord is 5 metres long; unstretched height of jump is 20 metres; the initial velocity v_0 at damping is -9.90 m/s; damping force is immediate at 5 metres; and the soldier with equipment is 100 kg [2]. Assumptions are: negligible air resistance due to a relatively small scale system; constant damping force throughout the fall; and as the cord is strapped to the soldier's waist, the modelled fall will stop at height of the soldier's leg which was reasonably assumed to be 0.8 metres long.

Results

Damping Coef. c	**Spring Constant** k
99.09 Ns/m	24.55 N/m
Impact Time t_f	**Final Velocity** v_f
5.89 s	0.54 m/s

Table 1: Final values

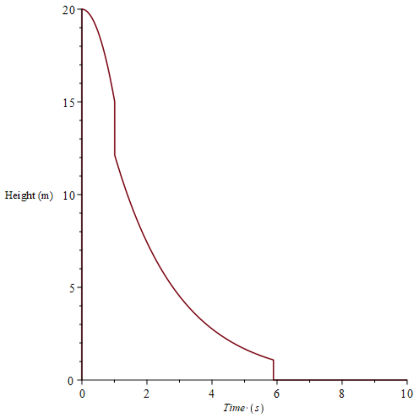

Figure 1: Modelled Bungee Fall of Soldier

Conclusion

We were successful in finding suitable value of k and c. When comparing to a typical spring constant for a bungee cord, 15 N/m [3], we achieved a stiffer spring constant within the same order of magnitude. This was expected as the system is being critically damped, so we would expect a greater damping force than in the under damping regime, so therefore there would be a stiffer spring constant. The soldier's 20 metre jump took place over a quick time of 5.89 seconds, with their final velocity being extremely low value of 0.54 m/s, indicating a safe descent, relating to a final kinetic energy of 14.58 J; this energy is negligible when comparing to the gravitational potential energy they would have, even at a height of 1 metre. With these finding we are able to conclude that our damped bungee system (theoretically) works for a fast, quiet and safe descent for a solider during a rappel.

References

[1] Gsu.edu.(2019).Damped Harmonic Oscillator.[online] Available at: `http://hyperphysics.phy-astr.gsu.edu/hbase/oscda.html`.(Date Last Accessed 06/10/23)

[2] The Soldier's load historical data of equipment - (Date last accessed 29/11/23)

[3] Bungee Jumping `https://stokes.byu.edu/teaching_resources/bungee.html` (Date Last Accessed 29/11/23)