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A5 1 Engulfing the Earth

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Abstract

In this paper we calculate the number of cows that, if the Sun was replaced with a black hole of equal mass, would be needed to be sent into the black hole for the black hole to engulf the Earth. We find that 1.007×10^{35} cows are required to increase the Schwarzschild radius such that the Earth is swallowed by the black hole, which is many orders of magnitude greater than the number of cows on Earth, making this an astronomically impractical method of destroying the Earth.

Introduction

In our scenario, a group of evil geniuses have plotted to destroy the Earth. They have so far managed to replace the Sun with a black hole of equal mass, however they were unsatisfied with this as it did not produce a change in the Earth's orbit and, furthermore, the black hole is too small to engulf the Earth unless extra mass is added to the black hole. In this paper, we calculate the number of cows that would need to be sent inside the black hole for the black hole to grow large enough to engulf the Earth.

Calculations

We assume the Earth's orbit is circular with a radius of 1 astronomical unit (1 AU), which is equal to 1.496×10^{11} m). This means the black hole has engulfed the Earth when the Schwarzschild radius of the black hole is greater than or equal to the radius of Earth's orbit. The Schwarzschild radius r_s of a black hole is given by the equation: [1]

$$r_s = \frac{2GM}{c^2} \quad (1)$$

Taking the gravitational constant G to be $6.674 \times 10^{-11} \text{ N m}^{-2} \text{ kg}^{-2}$ and the speed of light c to be $2.998 \times 10^8 \text{ m s}^{-1}$, we rearrange equation (1) for the mass of the black hole M :

$$M = \frac{r_s c^2}{2G} \quad (2)$$

and find that the mass of a black hole with a Schwarzschild radius of 1 AU is 1.007×10^{38} kg. This is seven to eight orders of magnitude larger than the mass of the Sun ($\approx 2 \times 10^{30}$ kg) and of a comparable mass to that of a supermassive black hole in the centre of a large galaxy. This means that we can neglect the initial mass of the black hole (one solar mass) in our calculations as it is much smaller than the total mass of the cows sent inside the black hole. Assuming that spherical cows with masses m equal to 1000 kg are sent into the black hole, we find the number of cows that would need to be sent inside the black hole by dividing M by m , giving an answer of 1.007×10^{35} cows.

Considering an elliptical orbit

In reality the Earth's orbit is elliptical, however we simplified this to a circular orbit in our

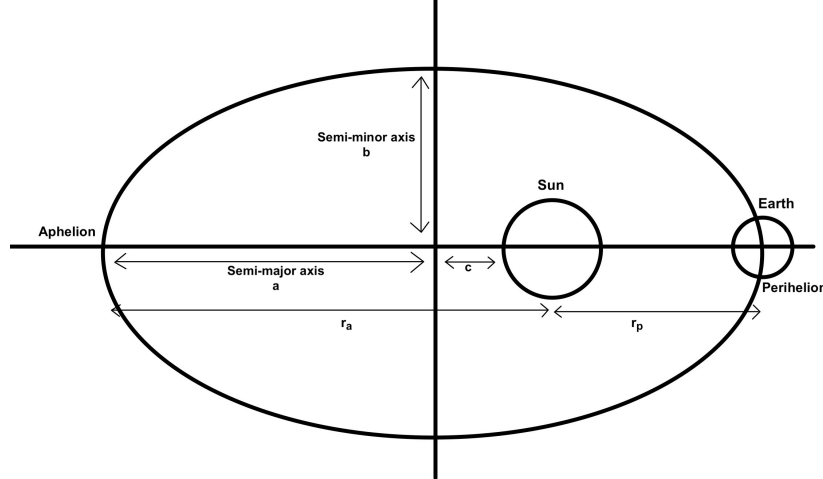


Figure 1: A diagram showing the orbital mechanics of the Sun-Earth system. Diagram not to scale.

calculations above to avoid having to take into account the position of the Earth in its orbit; when the Earth is at aphelion, the Schwarzschild radius of the black hole would need to be greater to engulf the Earth than when the Earth is at other points in its orbit. If the Earth's orbit is instead assumed to be elliptical with a semi-major axis a of 1 AU and eccentricity e of 0.0167 [2], we can calculate the distance c between the semi-major axis of Earth's orbit and the location of the black hole by multiplying the values of a and e together to give the value of c as 2.498×10^9 m. We can then calculate the aphelion and perihelion distances of Earth's orbit by adding and subtracting a and c , respectively, giving the aphelion distance $r_a = a + c = 1.521 \times 10^{11}$ m and the perihelion distance $r_p = a - c = 1.471 \times 10^{11}$ m. We can then find the difference in mass between the black holes with Schwarzschild radii at Earth's aphelion and perihelion by substituting the aphelion and perihelion distances into the Schwarzschild radius equation and subtracting the results from each other, giving a mass of 3.364×10^{36} kg which is equivalent to 3.364×10^{33} cows. This difference in mass is approximately 3% of the mass of the black hole with a Schwarzschild radius of 1 AU, which is not significant enough to affect the overall total number of cows being sent into the black hole.

Conclusion

We have found that the number of cows required to increase the Schwarzschild radius of the black hole such that the Earth becomes engulfed by the black hole is 1.007×10^{35} . This is many orders of magnitude larger than the number of cows on Earth, which was estimated as 1.5 billion in 2018 [3]. Many galaxies' worth of cows would be required to be sent into the black hole. Thus, this method of destroying the Earth would be astronomically impractical to achieve.

References

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