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A4_12 Relativity in the Christmas Spirit

A. Sier, R. Coulson, J. Wayman, H. Shehzad, H. Nutall

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

In this paper the speed at which Santa's sleigh travels is investigated with a focus on the relativistic effects. It is found that Santa is not travelling at great enough speeds for time dilation and length contraction to have a significant effect and that the red-shift of Santa and his sleigh causes a $\Delta\lambda$ = 9 nm shift. This is not significant enough to change the colour of Santa to an observer. It is also found that in order for Santa to be shifted from green to red, he must travel at 0.271 c.

Introduction

Every year millions of children put a glass of milk and a cookie or mince pie out for Santa on his trip around the world. This is an impossible feat without taking into account Christmas magic, but can the numbers be applied to special relativity?

The Journey

Santa always travels by night. In order to maximise the amount of time he has to deliver presents he must travel from the timezone furthest ahead to the one furthest behind. This is from Howard Island (sunset at 18:29 local time) to the Gilbert Islands (sunrise at 05:44 local time), both in the Pacific ocean [1]. Taking time zones into account, this gives Santa 35 hours and 15 minutes ($t = 1.27\times10^5$ s) to complete his journey. Starting from Howard Island and then travelling to New Zealand and Australia, up to Japan and Asia, across to Africa then up to Europe before finishing his journey in the Americas, Santa travels around 5.10×10^8 km ($d = 5.10\times10^{11}$ m) [2]. From the use of Eq. (1),

$$v = \frac{d}{t}, \quad (1)$$

where d is the distance of Santa's journey delivering presents and t is the time available to do this, Santa's velocity, v, is calculated to be 4.02×10^6 m s^{-1}. This is assuming that he is constantly travelling and does not have to stop when delivering presents due to Christmas magic.

Length Contraction & Time Dilation

In order to calculate length contraction and time dilation we must first find the value of γ. From Eq. (2),

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (2)$$

where v is as already defined and c is the speed of light, we find γ to be 1.00009.

Since the length of Santa's sleigh is approximately 9.14 m (30 ft, taken as an average from reference [3]), it is not long enough for the γ value to have a noticeable effect. Furthermore, in the reference frame of an observer on the Earth's surface t is not great enough for the γ value to cause a time dilation effect in Santa's reference frame. Therefore, Santa cannot be considered through the lens of special relativity.

Red-Shift

To find the red-shift of Santa's sleigh we use Eq. (3),

$$\lambda' = \lambda\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}, \tag{3}$$

where λ is the wavelength of Santa's sleigh at rest, λ' is the red-shifted wavelength of Santa's sleigh and the other terms are as previously defined. Using the value of v calculated in the previous section, taking c as 3.00×10^8 m s$^-1$ and using a value for the wavelength of red light (as this is the colour Santa is usually depicted as) of $\lambda = 700$ nm [4] we get a value of $\lambda' = 709$ nm ($\Delta\lambda = 0.4$ nm). This means that even at the great speed at which Santa soars across the sky, he is still not travelling at a great enough speed to change colour to an observer on Earth towards which he is moving.

However, in contrast to the modern depictions of Santa which show him as red due to "a certain American soft drinks company", Santa was originally green [5]. So how fast would Santa need to travel for his coat to be red when travelling and green when stationary? From rearranging Eq. (3) we get Eq. (4),

$$\frac{v}{c} = \frac{\lambda'^2 - \lambda^2}{\lambda'^2 + \lambda^2}. \tag{4}$$

From substituting $\lambda' = 700$ nm for red light and $\lambda = 530$ nm for green light (an average taken from reference [6]), we get a value for $\frac{v}{c} = 0.271$. This is a velocity of 8.13×10^7 m s^{-1}, so Santa would need to travel 20 times faster in a direction away from an observer for the American soft drinks company to be accurate.

Conclusion

From finding $\gamma = 1.00009$ it is clear that time dilation and length contraction would have no significant effect on Saint Nick's journey around the world. Also, from finding the red-shift of Santa we can see that he does not change colour as he travels towards an observer. However, if we flip the question around and find what speed Santa would need to travel for him to be perceived as red whilst actually being green we find that he needs to travel 20 times the speed we calculated from a distance time relation.

References

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