Journal of Physics Special Topics
An undergraduate physics journal

A5_9 Falling in the Rain

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December 5, 2022

Abstract

The goal of this paper was to investigate what effects rain might have on the terminal velocity of a person. By considering the exchange of momentum from collisions with raindrops and the increase of mass due to wet clothes, it was found that falling in the rain increased a persons terminal velocity by 0.8%.

Terminal velocity

The goal of this paper is to explore what effects the presence of rain has on a person's terminal velocity. To do that we must first establish what ordinary terminal velocity is.

Terminal velocity is the highest velocity an object can have while falling, and is achieved when the objects gravitational force (F_g) is equal to the force of air resistance (F_d). The force of gravity (N) is given by $F_g = mg$ where m is the mass of the object (kg), and g is acceleration due to gravity which we treat as a constant 9.81 ms^{-2}. As explained in [1] the force of air resistance (N) is given by $F_d = 0.5C_d\rho Av^2$. C_d is the dimensionless drag coefficient. ρ is the air density treated as a constant 1.225 kgm^{-3}. A is the area (m^2) of the surface colliding with the air. v is the velocity (ms^{-1}) the object is moving at.

Equating these two forces and rearranging for v provides the following equation for terminal velocity.

$$v = \sqrt{\frac{2mg}{C_d\rho A}}, \tag{1}$$

From [2] the ideal body weight for a male of height 177.8 cm is 71.00 kg. If a person is falling horizontally their frontal surface area can be found by assuming it is equal to a rectangle. Using the height mentioned previously, and a width equal to the average shoulder width of 41.10 cm [3], the average frontal surface area of a human was found to be 0.7290 m^2. The coefficient of drag for a human falling horizontally is effectively equal to 1 [4]. Using all of these values in (1) the terminal velocity for an average person falling without rain is 39.50 ms^{-1}

Including rain

With the standard terminal velocity found we then considered what effects rain would have on this model. The first effect comes from the force exerted due to collisions with the rain drops. We assumed that the raindrops are falling vertically at their terminal velocity, and that the impacts of those rain drops can be treated as inelastic collisions in which the rain drop sticks to the person. Doing so means that the force exerted by the rain on the falling person can be found by conservation of momentum. The momentum P (kgms^{-1}) of a raindrop is given by $P = m_r\Delta v$, where m_r is the mass of a raindrop (kg), and Δv is the difference in velocity between the raindrops (v_r) and the person. This means that the force experienced due to the rain (F_r) is given by $F_r = NP$. N is the number of rain drops colliding with the person each second (s^{-1}).

The second effect the rain will have will be to increase the mass of the person as they stick to them. This increase in mass (Δm) is given by, $\Delta m = N m_r t$. Where t is the time (s) spent falling in the rain.

The effects of rain can be combined with F_g and F_d to give the following expression for the total force (F) experienced while falling in the rain.

$$F = (m + \Delta m)g + F_r - F_d \tag{2}$$

The reason for F_d being negative is that the drag force acts in the opposite direction to gravity. F_r will initially be positive but will switch to being negative once the person's velocity exceeds that of the rain. This is accounted for in the Δv term.

Solving

To solve this we first had to find values for the various constants listed previously. [5] provided values for m_r and v_r of 34 x 10^{-6} kg and 9 ms^{-1} respectively. It also provided the radius of a raindrop 2 x 10^{-3} m, which was used to find a volume of 3.351 x 10^{-8} m^3, assuming the raindrops are spherical. The value of N was found by calculating the average number of raindrops that land in an area each second. According to [7] heavy rain has a depth of 4 mm per square meter per hour. Dividing by the volume of a raindrop mentioned previously, and the number of seconds in an hour obtains the number of raindrops that fall per metre squared per second. This can then be multiplied by A to find the number of raindrops expected to collide with a person each second. This value was found to be 24.17 s^{-1}.

Even with all the constants, equation (2) cannot be solved analytically due to the increase in mass varying with time. We therefore solved it numerically by simulating the previously described forces using python. The simulation calculated acceleration and velocity over time using (2), and stopped once the acceleration reached 0 ms^{-2}. At first, it was found that the constantly increasing mass meant there was no terminal velocity.

To correct for this, we introduced a limit to the mass by assuming that the maximum amount of water that can stick to a person is equal to the maximum amount of water their clothes can absorb. According to [7] clothes can hold up to their own weight in water. We measured the weight of an average males clothing to be $\sim$ 1.15 kg. This means the maximum mass was found to be 72.15 kg assuming their clothes were included in their standard weight. With this our simulation found that the terminal velocity of a person in the rain was 39.81 ms^{-1}.

Conclusion

The terminal velocity of a person falling in the rain was found to be 39.81 ms^{-1}, which was a 0.8% increase from the standard terminal velocity of 39.50 ms^{-1}. For comparison according to (1) this change in velocity could be matched by $\sim$ 1.15 kg of extra weight. This means the effects of rain drop collisions are largely negligible and the increase can largely be attributed to the extra weight from the wet clothes.

References

[1] `https://www.engineeringtoolbox.com/drag-coefficient-d_627.html` [Accessed 4 November 2022]

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