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A4_7 Spinning Worlds

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Abstract

In this paper the way in which the centripetal force changes over latitude is investigated for the different planets in our solar system. It is found that Venus has the smallest centripetal force, while Earth has the largest centripetal force. For all the planets the centripetal force is found to be at a maximum at the equator and decreasing to a minimum at the pole.

Introduction

The aim of this paper is to investigate how the centripetal force changes with latitude on different planets. The planets in our solar system have varied rotational periods and radii. Planets are not perfectly spherical in shape as their rotation causes matter to be pushed out at the equator causing their equatorial radius to be larger than their polar radius. Consequently, in this paper, the planets will be modeled as oblate spheroids rather than spheres.

Equations

The spheroid that the planets are modelled on is formed by rotating an ellipse around its semi-minor axis. This means that the equation for an ellipse [1] can be used to give the equation for a spheroid. The equation for the radius of a spheroid, R_s, at a latitude given by the angle, θ, from the semi-major axis of the ellipse, which is also the equatorial radius of the planet, R_e, is:

$$R_s = \frac{R_e}{\sqrt{R_e^2 sin^2(\theta) + R_p^2 cos^2(\theta)}} \tag{1}$$

Where R_p is the semi-minor axis of the ellipse, and is also the polar radius of the planet. The equation for the radial distance from the rotational axis to the planets surface, R_a, is:

$$R_a = R_s cos(\theta) \tag{2}$$

The equation for the radial distance from the rotational axis, R_a, can then be found by substituting equation (2) into equation (1) and is:

$$R_a = \frac{R_e R_p}{\sqrt{R_e^2 sin^2(\theta) + R_p^2 cos^2(\theta)}} cos(\theta) \tag{3}$$

The equation for the centripetal force, F_c, for an object of mass, m, rotating with a period, T, at a radial distance from the rotational axis, R_a, (as calculated in equation (3)) is:

$$F_c = m R_a \left(\frac{2\pi}{T}\right)^2 \tag{4}$$

Results

Data for the planets from the individual fact sheets found in [2] were used to find the centripetal force, F_c, per unit mass at different latitudes using equation (3) and equation (4). For Mercury, Venus, Earth and Mars the polar and equatorial radii were taken as the radius for the

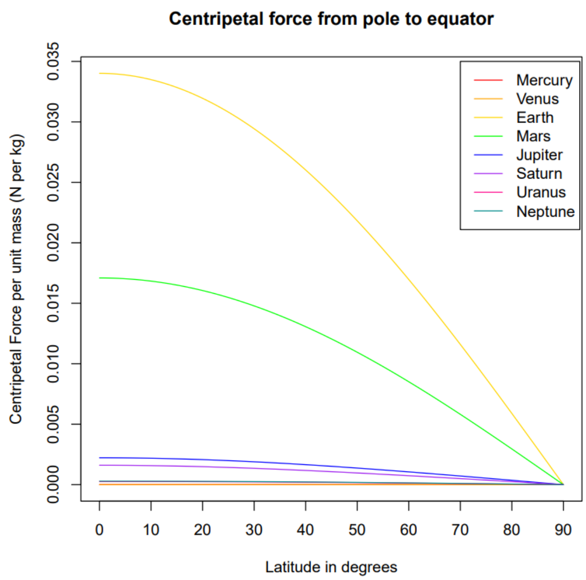

Figure 1: The centripetal force per unit mass of the different planets between latitudes of 0° to 90°.

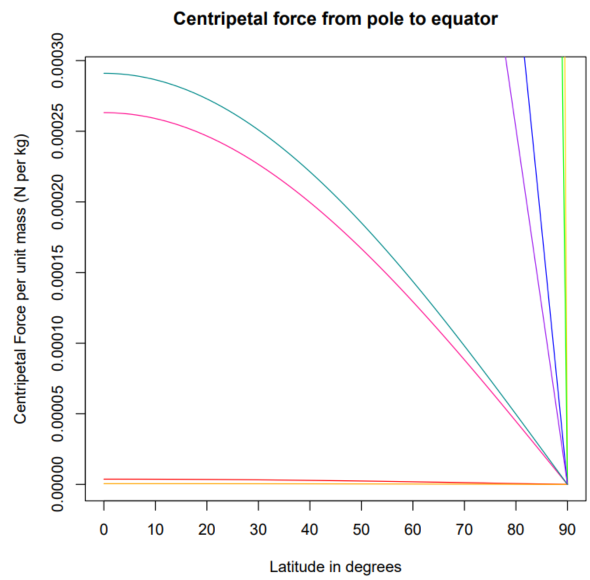

Figure 2: The planets with a centripetal force per unit mass below 3×10^{-4} N kg^{-1}.

planets surface. For Jupiter, Saturn, Uranus and Neptune the polar and equatorial radii were taken as the radius for the 1 bar pressure level as they do not have a solid surface. Figure (1) shows how the centripetal force changes with latitude for all the planets in the solar system. Many of the planets have a centripetal force per unit mass below 0.005 N kg^{-1} at the equator meaning that they appear very similar on the graph. Only Earth and Mars have a centripetal force per unit mass above this value.

Discussion

Planet	centripetal force per unit mass in N kg^{-1}
Mercury	3.75×10^{-6}
Venus	5.42×10^{-7}
Earth	3.40×10^{-2}
Mars	1.71×10^{-2}
Jupiter	2.22×10^{-3}
Saturn	1.60×10^{-3}
Uranus	2.63×10^{-4}
Neptune	2.91×10^{-4}

Table 1: The centripetal force per unit mass at the equator on the different planets.

The maximum centripetal force on each planet occurs at a latitude of 0°, the equator. The centripetal force per unit mass at the equator is given by Table (1). The minimum centripetal force on each planet occurs at a latitude of 90°, the pole. On all the planets the centripetal force per unit mass at the pole is 0 N kg^{-1}.

Conclusion

In conclusion all the planets have a maximum centripetal force at their equator with Earth having the largest per unit mass at 3.40×10^{-2} N kg^{-1} while Venus has the smallest per unit mass at 5.42×10^{-7} N kg^{-1}. All the planets have a minimum centripetal force at their poles with the centripetal force per unit mass of all the planets being 0 N kg^{-1}.

References

[1] `https://math.stackexchange.com/questions/432902/` [Accessed 21 October 2022]

[2] `https://nssdc.gsfc.nasa.gov/planetary/factsheet/` [Accessed 21 October 2022]