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P6_5 Simulating Charged Particle Motion in Electromagnetic Fields using Numerical Methods

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Abstract

We investigated the motion of a charged particle within an electromagnetic field using both the Euler and Boris methods. It has been shown that if the electric and magnetic field are perpendicular to one another then a charged particle will follow a cycloidal motion. We have also shown that the Euler method breaks down in the presence of a magnetic field and conclude that the Boris method is superior for simulating charge particle motion in fields.

Introduction

The motion of charged particles within electric and magnetic fields is well understood. We know that charged particles within an electric field will be accelerated in the direction of the field if they're positively charged and in the direction opposing the electric field if they're negatively charged. Similarly, we know that charged particles will gyrate around the field lines when placed in a magnetic field. We look to show what happens to the motion of charged particles when they're placed in a combination of electric and magnetic fields.

Numerical methods

We shall use two separate numerical methods to simulate the motion of a charged particle placed in an electromagnetic field(EM field). The first being the Euler method [1] which can be used to update the position of a charged particle with time. We initially define both the electric field($\mathbf{E}$) and magnetic field($\mathbf{B}$) and give our charged particle some initial velocity($\mathbf{v}$), charge(q), mass(m). This then allows us to determine the acceleration of the particle using equation 1.

$$\mathbf{a} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1}$$

We used the following normalised values to allow for simpler computation: $\mathbf{E}$ = [1,0,0] , $\mathbf{B}$ = [0,1,0] , $\mathbf{v}$ = [1,0,0] , $q = 1$, $m = 1$. We defined all vectors using arrays as this allows for the directions of vector quantities to be expressed.

From the acceleration we can calculate the position of a particle after time dt using the below expressions where the particle was initially positioned at the origin($x = 0$, $y = 0$, $z = 0$):

$$\mathbf{v} = \mathbf{a}dt \tag{2}$$

$$\mathbf{s} = \mathbf{v}dt \tag{3}$$

Equation 2 gives the velocity of the particle in passing from its initial position to its updated position($\mathbf{s}$) at time dt. Using the velocity we can calculate the updated position of the particle via equation(3). We repeat this process by plugging in our new initial velocity at the updated position into equation 1. Iterating through equations 1,2 and 3 for however many individual time steps(dt) we choose, allows us to map the position of the charged particle. The values for the positions at each time step are then stored in an array, which is used to graph the trajectory of the particle. The second method we shall use is the Boris method which follows the same idea as the Euler method however, the updated velocity at a given time step is calculated from a sequence of equations outlined in [2], which we referred to when producing our code for simulating the motion of a charged particle according to the Boris method.

Results and Discussion

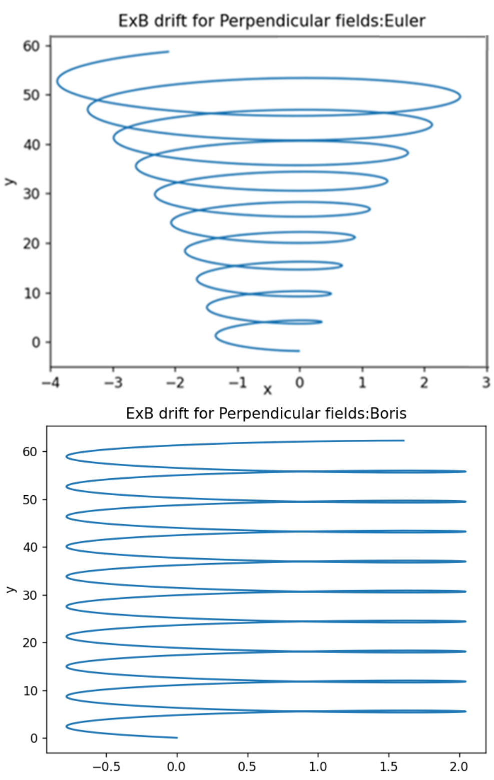

Figure 1: Motion of a charged particle in perpendicular E and B fields using Euler method(top) and Boris method(bottom)

In figure 1, we show the 2D trajectory of a charged particle according to both the Euler and Boris methods where the axis show the vertical and horizontal displacement. We used a time step of $dt = 0.05$ for a total of 1200 iterations and orientated the electric field ($\mathbf{E}$) to be in the x direction and the magnetic field ($\mathbf{B}$) to be going into the page. We can see that the charged particle follows a cycloidal motion, with the particle being accelerated by the electric field and forced to gyrate due to the magnetic field.

We observe a difference in the two plots as the Euler method shows the charged particle spiralling outwards throughout its trajectory which would indicate that the particle is gaining energy as for its gyro radius to increase the velocity of the particle must increase as the charge, field strength and mass were kept constant in this scenario. We know that the magnetic field does no work on a charged particle and only changes the direction of its velocity therefore the top plot does not correctly represent the motion meaning that the Euler method has broken down due to the magnetic field component of the motion.

Conclusion

Using both Euler and Boris methods, we have plotted the motion of a charged particle when both fields are perpendicular to each other and have shown that the charged particle exhibits cycloidal motion. In addition, we have shown that the Euler method breaks down when predicting the magnetic field component of the motion. Hence, we conclude that the Boris method is the superior method to use when simulating the motion of charged particles in EM fields.

References

[1] T. Sadler, E. Bates, L. Brewer, K. Smith *P3_7 Charged particles in a magnetic field - an integrator*, PST 21, (2022).

[2] `https://www.particleincell.com/2011/vxb-rotation` [Accessed 21 October 2022]