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Abstract

The aim of this paper is to determine the physics behind travelling to the bottom of the Marianas Trench in an enclosed hollow steel sphere. It was calculated that for an outer radius of 1m, the sphere requires a wall of thickness 15.7 cm to withstand the pressure and would weigh an estimated 13.1 metric tonnes.

Introduction

Travelling to the bottom of the ocean requires a vessel that can withstand the immense pressure that arises from increased depth under the surface. The deepest part of the ocean lies in the Mariana Trench, in a region known as Challenger Deep, with a depth of 10984 m [1]. The pressure at these depths is given by

$$P = \rho g H \tag{1}$$

where ρ is the density of water, g is the acceleration due to gravity, and H is the depth under the surface. This neglects the bulk modulus compression of water as its effect is negligible.

Theory

One method of travelling to great depths in the ocean has been a thick-walled, hollow steel sphere with enough internal space for a passenger to travel inside, such as the "bathysphere" [2]. The water outside will create an external pressure pushing on the walls of the sphere, whilst the air inside the sphere will push outwards and create an opposing pressure. The pressure inside the sphere is assumed to be standard atmospheric pressure (∼0.1 MPa) which is assumed to be negligible. A diagram depicting the above is shown in Fig. 1.

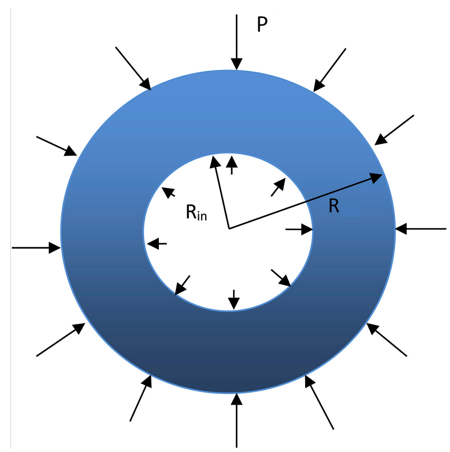

Figure 1: Cross-Sectional View illustrating the negligible internal pressure and the large external pressure forces acting on the hollow steel sphere. Image from [3]

Due to the symmetry of the sphere, the stress in all locations must be the same meaning there is no shear stress across the vessel [4]. The forces acting on the sphere can therefore be calculated by evaluating only one half of the sphere. The horizontal component of the hemisphere will experience both a tensile force and a fluid force.

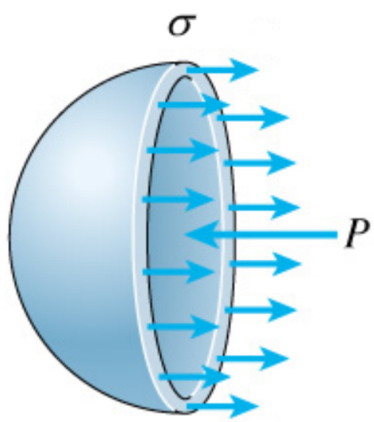

Figure 2: Image showing the horizontal forces acting on the hemisphere. Image from [4].

Both tensile and fluid forces in the horizontal direction across the hemisphere have no net force:

$$F_{\text{net}} = F_{\text{tension}} + F_{\text{fluid}} = 0 \tag{2}$$

The tension force, F_{tension}, is given by tensile strength, σ, multiplied by surface area of the hemisphere, A_{S}:

$$F_{\text{tension}} = \sigma A_{\text{S}} \tag{3}$$

Surface area of the hemisphere is given by:

$$A_{\text{S}} = (2\pi R)t\,, \tag{4}$$

where t is the thickness of the sphere. The fluid force is given by the cross sectional surface area multiplied by the external pressure:

$$F_{\text{fluid}} = -PA_{\text{cs}} = -P\pi R^2 \tag{5}$$

Therefore, the required wall thickness is found by substituting Eq. (4) into Eq. (3), and the results of this and Eq. (5) into Eq. (2).

$$t = \frac{P\pi R^2}{2\pi R\sigma} = \frac{PR}{2\sigma} \tag{6}$$

The mass of the sphere can be found using:

$$M = \rho_{\text{St}} V_{\text{St}} = \rho_{\text{St}} \frac{4\pi}{3}\left(R^3 - R_{in}^3\right) \tag{7}$$

where the inner radius is outer radius minus wall thickness, $R_{in} = R - t$, and ρ_{St} and V_{St} are the density and volume of steel respectively. Due to Eq. (6), increasing the sphere's radius increases the required wall thickness, thereby increasing the mass and costs of materials to produce the sphere. The radius was therefore chosen to be $R = 1$ m to save on costs, but to still have enough internal space for a single passenger.

Results and Discussion

Using the density of sea water near the Mariana Trench as ≈1022 kg m^{-3} [5] in Eq. (1), the pressure is calculated to be $P \approx 110$ MPa. Steel has a yield tensile strength and density of $\sigma_{\text{St}} = 350$ MPa and $\rho_{\text{St}} = 7800$ kg m^{-3} respectively [6]. Therefore, the walls are required to be 15.7 cm thick when the above values are substituted into Eq. (6). The inner radius will therefore be $R_{in} = 84.3$ cm and the mass of the sphere will be 13.1 metric tonnes.

In practicality, the thickness of the walls would be required to be thicker by a safety factor, and the density of the sphere will be much greater than that of the surrounding water so it will remain at the bottom of the trench to the detriment of the passenger inside. A future paper might evaluate the possibilities of transporting the sphere in a round trip from the surface to the bottom and back agaim.

References

[1] J. V. Gardner, A. A. Armstrong, B. R. Calder, and J. Beaudoin (2014) So, How Deep Is the Mariana Trench?

[2] `https://bit.ly/3G1N5pL` [Accessed 30 November 2021]

[3] `https://bit.ly/3xDDdiS` [Accessed 30 November 2021]

[4] `https://bit.ly/3o8jNPY` [Accessed 30 November 2021]

[5] P. Webb *Introduction to Oceanography.* (Pressbooks, 2021), Vol. 1, Chapter 6.3

[6] `https://bit.ly/31avqwV` [Accessed 30 November 2021]