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P2_9 Leap Year? Get out of Here! Part 1

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Abstract

In this paper we investigate the feasibility of altering the Earth's orbit, in order to remove the leap year. Since the Earth orbits the Sun with a period of 365.25 days, the orbital period must be reduced by 0.25 days. Under the assumption that the Earth's orbit is circular at all times, we find that 1.22×10^{30} J of energy must be removed from the orbit, and if this energy is to be removed over the course of one orbit, the acceleration of the Earth must be 2.17×10^{-7} ms^{-2}. Removing this energy will reduce the radius of the Earth's orbit by 0.046%.

Aim

A single calendar year on Earth is 365 days, but the Earth's orbital period around the Sun is 365.25 days[1]. To account for this, every fourth calendar year is a "leap year", which has an extra day at the end of February. In order to remove the leap year, one would have to reduce the Earth's orbital period by 0.25 days, so that each orbit matches up exactly with one calendar year. *Note: throughout this paper we assume the Earth's orbit is always circular for simplicity.*

Period Reduction and Semi-Major Axis

If the Earth's current orbital period is T_1, and the period we want is T_2, then the orbital period reduction of 0.25 days is given by:

$$\Delta T = T_1 - T_2 = 0.25 \text{ days} = 21600 \text{ s}. \quad (1)$$

One can write the orbital period in terms of the gravitational constant G, the solar mass $M_\odot = 2 \times 10^{30}$ kg[2], and the semi-major axis r using Kepler's Third Law[3]:

$$T = \sqrt{\frac{4\pi^2}{GM_\odot} r^3}. \quad (2)$$

If r_1 is the initial semi-major axis, and r_2 is the semi major axis of an orbit without a leap year, one can rewrite Eq. (1) as:

$$\Delta T = \sqrt{\frac{4\pi^2}{GM_\odot} r_1^3} - \sqrt{\frac{4\pi^2}{GM_\odot} r_2^3} = 21600 \text{ s}. \quad (3)$$

Since $r_1 = 1.49598023 \times 10^{11}$ m and the gravitational constant is known ($G = 6.67 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} [4]), one can solve for the semi-major axis of an orbit with no leap year, $r_2 = 1.495295771 \times 10^{11}$ m (the large number of significant figures is necessary as the orbit is only changed by a small amount).

Energy Change

Using r_1 and r_2 we can begin to describe the initial and final orbits in terms of their total energy. An orbit has a potential energy U given by Eq. (4) and a kinetic energy K given by Eq.

(5), where $m = 5.972 \times 10^{24}$ kg is the mass of the Earth[2] and v is the tangential velocity of the Earth.

$$U = -\frac{GM_{\odot}m}{r}, \tag{4}$$

$$K = \frac{1}{2}mv^2. \tag{5}$$

The velocity of a circular orbit can be computed from the *Vis-Viva Equation*, simplified for the circular orbit case[3]:

$$v = \sqrt{\frac{GM_{\odot}}{r}}. \tag{6}$$

Thus the total energy for the initial and final orbits can be found by summing eqs. 4 and 5, incorporating Eq. (6) and substituting in known values.

E_1	$-2.662684921 \times 10^{33}$ J
E_2	$-2.663903741 \times 10^{33}$ J

The energy difference between the two orbits, ΔE, is given by $E_2 - E_1$.

$$\Delta E = -1.21882 \times 10^{30} \text{ J}. \tag{7}$$

Doing Work on the Orbit

To remove the leap year, one needs to remove energy from the Earth's orbit. We assume some process exists which can apply a force, F, in the opposite direction to Earth's tangential velocity. Applying such a force against the Earth's motion will do work, W, on the orbit, lowering its total energy. The definition of work is force multiplied by distance:

$$W = Fd. \tag{8}$$

Making the reasonable decision to apply this force for an entire orbit[1], the distance is the orbital circumference, $d = 2\pi r$. Since $r_1 \approx r_2$, we assume $r = r_2$ as this gives the maximum force that would need be applied. Thus to do 1.22882×10^{30} J of work in one orbit, one would need to apply a force of:

$$F = \frac{W}{2\pi r_2} = 1.2973 \times 10^{18} \text{ N}. \tag{9}$$

Acceleration

From Newton's Second Law,

$$a = \frac{F}{m}, \tag{10}$$

one can calculate the acceleration felt by the Earth due to the calculated force. Even though the force is very large, the Earth's mass is also large, so the acceleration felt would be only 2.17 $\times 10^{-7}$ ms^{-2}. Compared to the acceleration felt due to Earth's gravity, this would be practically imperceptible.

Summary

The Earth's orbital period could be reduced by removing 1.21882$\times$ 10^{30} J of energy from its orbit, leading to the leap year no longer being required. A force acting opposed to the Earth's orbital motion would do work on the orbit, reducing the orbital energy by the required amount after one orbital revolution if the force was equal to 1.2973×10^{18} N. This would accelerate the Earth by a nigh-imperceptible 2.17×10^{-7} ms^{-2}.

References

[1] SI Units, Table 5, International Astronomical Union `https://www.iau.org/publications/proceedings_rules/units/` Retrieved 07/12/2021

[2] *The Astronomical Almanac* (United States Naval Observatory, 2014)

[3] R. Bate, D. Mueller, J. White, *Fundamentals of Astrodynamics* (Dover Publications Inc., New York, 1971) p.33 - 34

[4] R. Bate, D. Mueller, J. White, *Fundamentals of Astrodynamics* (Dover Publications Inc., New York, 1971) p.4

[1] Reasonable because of our previous assumption that the orbit remains circular at all times. If the force were to be applied only on one segment of the orbit, the final orbit would have to be an ellipse.