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P4_4 Temperature Check

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Abstract

In this paper, the time that a hot coffee would remain at a desirable drinking temperature when placed in a vacuum flask is calculated. The time for the coffee's temperature to drop from 90°C to below a suitable drinking temperature of 50°C [1] was calculated to be 12.17 hours. The coffee is in a suitable drinking range of 60°C to 50°C for 4.29 hours.

Introduction

The zeroth law of thermodynamics states that the temperature of a medium will always try to reach equilibrium with its surroundings. This can be achieved in three ways: conduction, convection and radiation. The main method for keeping a liquid at a constant temperature is to use a vacuum flask. This is a vessel that has two containers, a glass case that has been made reflective by silvering it and a stainless steel outside for protection.

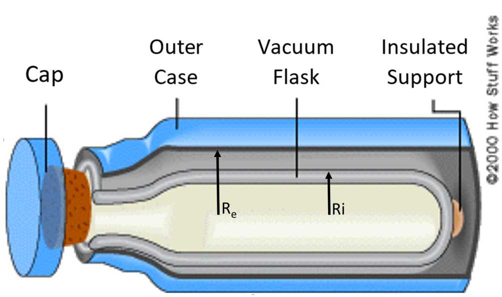

Figure 1: The cross-section of a thermos flask including the internal radius (R_i) and external radius (R_e) [2].

The silvering of the glass layer helps make the heat loss, due to radiation, negligible as it is reflected until the flask is opened. Between these two layers lies an evacuated air layer that acts very similarly to a vacuum, creating a medium with thermal conductivity of 5.0 $mWm^{-1}K^{-1}$ [3]. This results in temperature changes due to conduction. The temperature change due to convection is accounted for by the flask having an air-tight seal. This prevents the movement of outside air affecting the temperature of the liquid inside the flask.

Equations

The flask is assumed to be comprised of 2 uniform cylindrical containers which were measured to have an internal radius (R_i) of 3.23 cm and an external radius (R_e) of 3.60 cm. The height (L) and volume (V) of the internal cylinder are 18 cm and 590 ml [4]. This internal and external flask are also assumed to have a uniform separation distance (Δx). The contribution of the material holding the vacuum flask in place is assumed to be negligible. The liquid inside is also treated as water since coffee is mainly water. The heat lost through the lid will also be neglected. This means that heat loss to the surroundings will only be considered through the walls and bottom of the container giving a cross-sectional area (A) of:

$$A = 2L\pi r_i + \pi {r_i}^2, \quad (1)$$

The change in temperature due to conduction can be found by considering the energy required to increase the temperature of the liquid inside by one degree Kelvin (specific heat capacity) [6] and equating it to the thermal current through the vacuum-like medium.

$$\frac{dQ}{dt} = \frac{mcdT}{dt}, \quad (2)$$

Where the thermal conduction coefficient (K) is 5.0 $mWm^{-1}K^{-1}$ [3], cross-sectional area (A) is 0.040 m^2, the specific heat capacity (c) is 4200 $JKg^{-1}K^{-1}$ [5] and the change in temperature is dT. The equation below is the thermal current transmitted through a given surface area [6].

$$I = \frac{dQ}{dt} = -\frac{KA\Delta T}{\Delta x}, \quad (3)$$

The flask has a difference between R_i and R_e (Δx) of 0.37 cm, a cross-sectional area (A) of 0.040 m^2 and a mass (m) of 0.59 kg. The mass was found by multiplying the density of water (1 g/cm^3) [5] by the volume of the inner cylinder (V). When equating Eq. (2) and Eq. (3) and rearranging the following equation is given.

$$\int_0^t dt = -\frac{mc\Delta x}{KA}\int_{T_I}^{T_F} \frac{dT}{\Delta T}, \quad (4)$$

ΔT can be expressed as the difference between the initial (T_I) of 90°C or final (T_F) temperature and the temperature of the surroundings T_S which is assumed to be 25°C. This gives a final expression for the time taken for the liquid inside the flask to cool down.

$$t = \frac{mc\Delta x}{KA} \ln\left(\frac{T_I - T_S}{T_F - T_S}\right), \quad (5)$$

Results

Fig. 2 shows that the temperature falls off exponentially with time as the temperature of the liquid inside the flask approaches that of the surroundings. The graph shows that the coffee

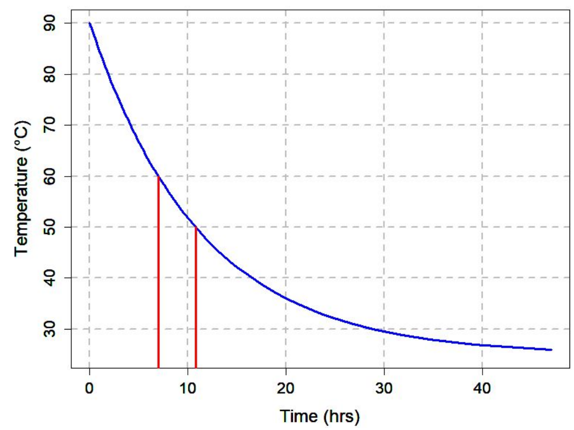

Figure 2: The temperature decay of the coffee inside the thermos flask as a function of time.

takes 7.88 hours to reach 60°C and (first red line) and 12.17 hours to reach 50°C (second red line).

Conclusion

The coffee remains between 60°C and 50°C which is a suitable drinking temperature [1] for 4.29 hours. When compared to a regular mug it takes approximately 45 minutes to reach 50°C [7]. This means that using a vacuum flask results in a 1523% increase in the time taken to reach the minimum acceptable temperature.

References

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