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P4_2 Earth's Toilet Trouble

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Abstract

How large a radius would a toilet bowl need to have in order to cause the Earth to stop rotating? Is this even possible? In this paper, a circular motion model of the common flush toilet is explored to answer these hypotheses. By keeping the height of the flush constant, we have calculated that the toilet bowl would need to have a radius of $8.29 \pm 0.41 \times 10^7$ m for the angular momentum to equal that of the Earth's. The velocity of the water was also calculated as $1.33 \pm 0.07 \times 10^8$ ms^{-1} which, although verifies this scenario is theoretically possible, is not feasible due to the vast nature of construction.

Introduction

The first version of the flushing toilet was invented in 1596 by the godson of Queen Elizabeth 1, Sir John Harington [1]. Although it may seem a mundane object, the vortex motion of the flush has some very interesting physical properties, most importantly angular momentum.

In determining whether a toilet of sufficient radius can cause the Earth to stop rotating, certain assumptions are made. It is assumed the Earth is perfectly spherical and the flushing vortex is conical, and that both it and the Earth have uniform mass. This allows the usage of the standard moment of inertia equations for each shape.

Moreover, only the maximum angular velocity of the toilet flush is used so the radius of the toilet is the only independent variable. Finally, the toilet is located at the North Pole ensuring the flushing motion, when the water begins its vortex motion to when it ceases, must occur in a clockwise direction to have an opposite force against the Earth's anti-clockwise rotation [2].

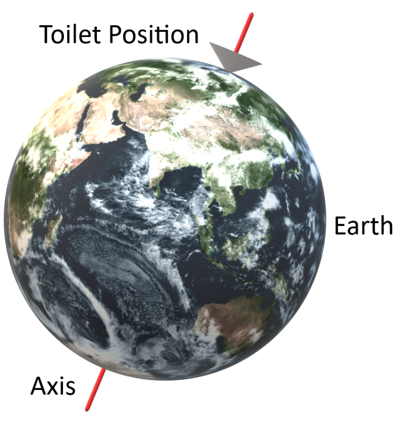

Figure 1: A diagram we created that shows the toilet position identified that causes the greatest momentum change for the Earth's rotation.

Throughout this paper, the effect of angular momentum generated from the toilet flush will be compared with that of the Earth and through graphing and numerical analysis, a critical radius, where the Earth's rotation about its axis ceases, will be determined.

Angular Velocity of an Average Flush

As we are assuming the angular velocity (ω) is constant, a value must be found:

$$\omega = \frac{\Delta\theta}{\Delta t}, \tag{1}$$

where $\Delta\theta$ is the change in angular displacement and Δt is defined to be the duration [3].

To determine the angular velocity, a weight on a string was placed into the bowl and the number of full revolutions that occurred were counted as well as timing the complete duration of the flushing motion. It was determined the duration of vortex motion is 5.00 ± 0.25s, the error due to human reaction time [4] and completed 8 revolutions; therefore $\omega = 1.60 \pm 0.08$ rev s^{-1} accounting for the error in duration.

Angular Momentum

Angular momentum (L) is calculated using:

$$L = I\omega, \tag{2}$$

where $I = \frac{2}{5}MR^2$ for a sphere [5] and $\frac{3}{10}MR^2$ for a cone [6]. M is the mass of the water which changes value depending on the radius R.

To calculate the mass of water, we assume the height (h) of the conical vortex is 0.30 m and:

$$M = \rho V, \tag{3}$$

where ρ is the density of water, 997 kgm^{-3} [7], and $V = \frac{1}{3}\pi R^2 h$ is volume for a cone [8].

Results

A radii sample is created and the angular momentum at each radius is calculated and plotted.

Using Fig. 2 and the angular momentum of Earth about its axis: 0.71×10^{34} kg m^{2}s^{-1} [5], it is possible to find the radius the Earth stops rotating at: $8.29 \pm 0.41 \times 10^7$ m. At this radius, the mass of the water is $2.16 \pm 0.11 \times 10^{15}$ kg.

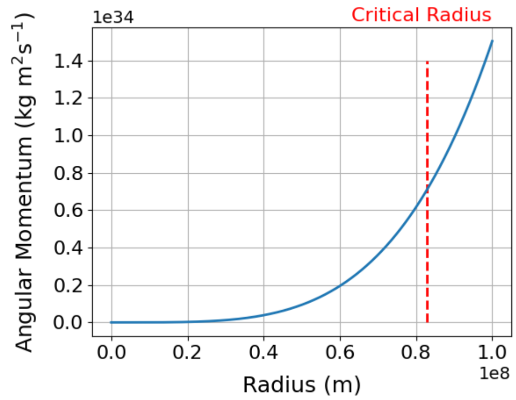

Figure 2: A plot of the flushing motion angular momentum against the radius of the toilet flush. Highlighted is the critical radius for when Earth's angular momentum = 0.

Conclusion

Using this simplified model, we determined flushing a toilet of radius calculated previously would halt the Earth's rotation once flushing concluded. To check if the velocity (v) of the water at the edge is theoretically possible to cause this scenario, we can use Eq. (4):

$$v = r\omega, \tag{4}$$

The water would be moving at a velocity of $1.33 \pm 0.07 \times 10^8$ ms^{-1}, $\times 0.44$ the speed of light which although theoretically possible, is impossible to replicate using current technology.

References

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