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A2_1 The Need for Speed

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Abstract

This article assesses the acceleration that a human could achieve on roller skates with different wheel shapes and slope inclinations. The maximum acceleration without propulsion down the world's steepest hill was calculated to be $6.01ms^{-2}$ and the extra force needed before they perished was found to be $10749N$, supplied in excess by 7 CESARONI 54-6GXL IMAX model rocket engines.

Introduction

All moving objects have kinetic energy, but rotating moving objects have an additional component of rotational energy with analogous equations to those of linear motion. This article explores the maximum acceleration that a skater on the world's steepest hill could reach and the extra propulsion needed to reach the maximum acceleration that a human could survive.

Theory

The moment of inertia for a rotating object varies depending on the object's shape. In general terms, the moment of inertia for any shape is given as

$$I = bmr^2, \tag{1}$$

where b is a dimensionless number determined by the shape of the object.

The total kinetic energy in the system, K, is the sum of the translational energy of the centre of mass (COM), $\frac{1}{2}mv^2$, and the rotational energy about the COM, $\frac{1}{2}I\omega^2$. Taking the mass of the roller skates and human combined as M, the individual wheel mass as m, the wheel number as N, the angular velocity of a wheel as ω, the velocity of the COM as v and the moment of inertia to be I, then the total kinetic energy is

$$K = \frac{1}{2}(M + Nm)v^2 + \frac{1}{2}NI\omega^2. \tag{2}$$

Under the condition that no slipping of the wheels occurs, we take $v_{COM} = r\omega$ and the kinetic energy formula becomes

$$K = \frac{1}{2}(M + Nm + N\frac{I}{r^2})v_{COM}^2. \tag{3}$$

The only force felt by the skater is the acceleration due to gravity, g. By taking this force contribution on an inclined slope of θ degrees, the total energy, E, in terms of kinetic and potential energy, U, of the skater is

$$E = K + U \tag{4}$$

$$E = \frac{1}{2}(M + Nm + \frac{NI}{r^2})v_{COM}^2 - (M + Nm)gd\sin(\theta) = 0 \tag{5}$$

where d is the rolling distance down the slope. By taking the time derivative of Eq.(5) and rearranging, the acceleration, a, of the skater is found to be

$$a = \frac{(M + Nm)g\sin(\theta)}{M + Nm(1 + b)}. \tag{6}$$

As such, the linear acceleration is dependent on wheel shape but not wheel size. Two extreme approximations for the shapes of the wheel are that they are either hollow cylinders of radius r with an axle through the middle, or they are solid, uniform cylinders with a central axle.

Results and Discussion

A discussion of how different wheels and inclinations affect the maximum acceleration reached by a skater follows.

We define the mass of a human and a pair of roller skates to be $70kg$ and $4kg$ respectively. Hence $M = 74kg$, with each wheel's mass being $m = 0.1kg$. We assume a standard number of wheels on a pair of roller skates of $N = 8$ and $g = 9.81ms^{-2}(2dp)$. The effects of friction and air resistance are ignored in this model.

Model 1: Imagine someone in roller skates is going down the steepest street in the world with inclination of 34.8% [1], then $\theta = \arctan(0.348) = 19.2°$.

When the shape of the wheels is analogous to hollow cylinders then $b = 1$, and when the wheels are solid cylinders $b = \frac{1}{2}$ [2]. Using Eq. (6), the accelerations are $a = 3.33ms^{-2}$ and $a = 3.35ms^{-2}$ respectively to 2 d.p.

This shows that while there is some dependence on the wheel shape, the type of roller skate wheels you pick will not make much difference to the acceleration you can reach. Either approximation of the wheels being hollow or solid will provide an appropriate estimate for the acceleration experienced by the skater since they have a difference of just 0.6%.

Model 2: A more extreme model would be to imagine skating down an incline more comparable to the steepest ski slope in the world at an inclination of 78% [3], hence $\theta = 38°$.

In this case the acceleration with hollow and solid wheels respectively would be $a = 5.98ms^{-2}$ and $6.01ms^{-2}$ in 2 d.p.

The human body has been tested to withstand approximately $20G$s of acceleration horizontally [4]. Even under the conditions of free fall a human could only accelerate to a maximum of 1G which they could survive over a long duration.

To find the additional thrust needed to accelerate the skater to $20G$s, we imagine attaching 7 CESARONI 54-6GXL IMAX model rocket engines each weighing $2.542kg$ and providing $1585.6N$ of thrust [5] are attached to the skater. Using $F = ma$ down an incline and taking the acceleration with which the skater is already travelling down the slope to be $6.0ms^{-2}(1dp)$, the additional acceleration needed to kill the skater is $190ms^{-2}$. Therefore, the force needed to accelerate the skater to deadly speeds is 10749N, which would be exceeded by the $11099N$ of force provided by 7 rocket motors.

Conclusion

It was found that acceleration was independent of wheel size, lightly effected by wheel shape and that 7 CESARONI 54-6GXL IMAX model rocket engines could propel a human down the world's steepest hill to deadly accelerations. Further work is advised to assess this result under more realistic conditions with appropriate friction and air resistance drag analysis included.

References

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