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A5_1 Charlie and the Fizzy Lifting Drink

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Abstract

In order to lift Charlie off the ground, he would need to consume approximately 2922 bottles of Wonka's helium filled soda (neglecting any heating of the gas by Charlie's body temperature), or approximately 2888 bottles if we consider heating of the helium and treat Charlie like a human balloon that can expand. This roughly equates to $5.99kg$ and $5.92kg$ of helium respectively.

Introduction

In Charlie and the Chocolate Factory, Wonka is creating a 'fizzy lifting drink'. Whilst Wonka's back is turned, Charlie and Grandpa Joe help themselves to the drink and begin to float up into the air. The only way back down is to burp to release the gasses. In reality, gasses that are less dense than air are called 'lifting gasses' and have the capability to lift mass in the air, such as helium balloons. We investigate how many bottles of soda Charlie would need to consume to be lifted up, if instead of being made with carbonates, the drink is made fizzy with helium (ignoring side effects of ingesting large quantities of helium). We have swapped the CO_2 usually used in fizzy drinks for helium, as CO_2 is denser than air at STP (standard pressure and temperature conditions) and lift is not generated.

Theory

We start by using a standard $330ml$ bottle of Coca-cola for reference for how much CO_2 is in a carbonated drink. There is $2.20g$ of CO_2 in one can ($355ml$) of cola [1], converting this for $330ml$ we take a ratio:

$$(\frac{355}{330}) \times 2.20g = 2.05g \tag{1}$$

Thus in our $330ml$ bottle there is approximately $2.05g$ of CO_2. We assume that to create the same effect of a fizzy drink, the same amount of helium is needed. Using buoyancy equations, we can determine the mass helium is capable of lifting per m^3 at sea level, with STP conditions (P= $101atm$ and $T = 15°C$). A volume, V, of a material (helium gas) with density ρ_{he} in a fluid of density ρ_{air} (air), will experience a buoyant force:

$$F_b = gV\rho_{he} \tag{2}$$

Where g is the acceleration due to gravity, $9.81ms^{-1}$. It also has weight:

$$W = -gV\rho_{he} \tag{3}$$

By adding (2) and (3) we get the final equation for the lifting force required:

$$F_l = gV(\rho_{air} - \rho_{he}) \tag{4}$$

Where the equation in brackets tells us the amount of mass helium is capable of lifting.

Results

The density of air is given as $1.225kgm^{-3}$ [2] and the density of helium as $0.179kgm^{-3}$ [3], and using Eq.(4) gives a value of $1.046kgm^{-3}$ for the lifting capability of helium (which we denote by $\rho_{he,lift}$). To calculate the volume of helium needed to lift Charlie, we take Charlie's mass, m_C, divided by $\rho_{he,lift}$. In the story, Charlie is 11 years old and healthy, thus his mass is approximately $35kg$ [4]. This gives $33.46m^3$ of helium needed to cause lift, and converting to mass is $5.99kg$. To work out how many bottles of soda this is, we divide the mass of helium needed by the amount of helium in each bottle, which is 2921.95 bottles or around 2922.

Nearly 3000 bottles is a very large number. Now we consider the effect of Charlie's body temperature heating up the helium, much like how air is heated in a hot air balloon to cause it to rise. We remain at sea level so the pressure is the same, and just the temperature, T, changes to body temperature, $37°C$ [5]. Using the Ideal Gas law we calculate the 'new' density of helium after being heated (assuming all helium is heated at the same rate and not dependent on the rate of consumption), the new lifting capability and the new number of bottles needed. The initial temperature and volume are T_1 and V_1 and the new are T_2 and V_2. As $PV = nRT$ and P remains the same at sea level, we can replace the T and V with the relevant forms and equate the two equations to solve for V_2, our only unknown.

$$\frac{nRT_1}{V_1} = \frac{nRT_2}{V_2} \tag{5}$$

We rearrange to give $V_2 = (V_1 T_2)/T_1$. Substituting in our values, ($T_1 = 15°C, T_2 = 37°C, V_1 = m_{he,atom}/\rho_{he}$) gives a V_2 value of 4.00 x $10^{-26}m^3$. Therefore, the 'new' density of helium is $0.166kgm^{-3}$. Using this, we get a value of $1.059kgm^{-3}$ for helium's lift capability. Thus, the volume of helium required to lift Charlie is $33.05m^3$ which is $5.92kg$. This translates to 2887.80 bottles of Wonka soda, or around 2888. Roughly, by heating the helium (and also expanding like a balloon as the helium heats, assuming he can do this), Charlie can only drink 34 bottles fewer of Wonka's lifting drink. Note that this situation does not vary with height as Charlie is lifted. In the film, they appear rise around $10m$ in the air, thus STP conditions are the same. However, if we were considering how this would work if they were lifted much higher, i.e. where pressure and temperature change compared to sea level, the new P and T values would need to be considered.

Conclusion

Whilst it is possible that if the fizzy lifting drink was somehow made with helium instead of CO_2, Charlie would need to consume nearly 3000 bottles of soda, which remains true even when we consider how his body temperature can heat the ingested helium. We also consider that to return to the ground, an equal amount of gas would have to be expelled (i.e. burped out). This is around $6.00kg$ which would likely take a considerable amount of time, at which point Charlie could be extremely high up or could have even floated away.

References

[1] Chemistry Stack Exchange `https://tinyurl.com/a8avy5zt` [Accessed 6th October 2021]

[2] `https://www.engineeringtoolbox.com/air-density-specific-weight-d_600.html`[Accessed 6th October 2021]

[3] `https://www.aqua-calc.com/page/density-table/substance/helium`[Accessed 25th October 2021]

[4] RCPCH, *BOYS UK Growth chart 2-18 years*, (Jan. 13, 2012), Issue 2, `https://www.rcpch.ac.uk/sites/default/files/Boys_2-18_years_growth_chart.pdf`

[5] Hutchison JS et al; Hypothermia Pediatric Head Injury Trial Investigators and the Canadian Critical Care Trials Group. *Hypothermia therapy after traumatic brain injury in children.* N Engl J Med. 2008 Jun 5;358(23):2447-56.doi: 10.1056/NEJMoa0706930.