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P4_1 Earth on Tour

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Abstract

What would happen if the Earth experienced a series of coincidental velocity changes similar to that of a Hohmann transfer that aligned both the orbits of itself and it's planetary neighbour, Mars? In this paper, we consider this hypothetical scenario to have occurred and aim to model the velocity changes and forces required for this to occur. We found that the total change in velocity for this manoeuvre would be 6.843 kms^{-1} and total force on the Earth would be 4.087×10^{28} N.

Introduction

In 1925, Walter Hohmann published his groundbreaking paper on 'The Attainability of the Heavenly Bodies' [1] in which he outlines the theory that two velocity impulses applied to a spacecraft can cause a transfer from an initial coplanar orbit to another coplanar orbit of non-identical radius through an elliptical transfer orbit (ETO) [2].

For this investigation, two key assumptions have been made where both the Earth's and Mars' gravitational and magnetic fields do not interact with one another as well as modelling both planets as spheres of uniform mass. Moreover, quantitative values are to the accuracy of three decimal places.

Earth on the Move

To begin the calculations, several orbital parameters for both Earth and Mars must be found: the periapsis and apoapsis distances of the respective orbits [3][4] and the standard gravitational parameter using the Sun's mass [5].

When performing a Hohmann transfer, the ETO must have a semi-major axis (a_{trans}):

$$a_{trans} = \frac{r_{ip} + r_{fa}}{2}, \tag{1}$$

where r_{ip} is the initial orbits periapsis distance and r_{fa} is the final orbits apoapsis distance [6].

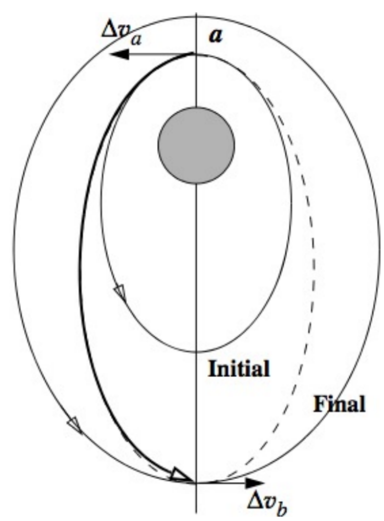

Figure 1: Diagram of the Earth (**a**) completing a Hohmann transfer from its initial elliptical orbit to a final orbit mirroring that of Mars [6]. (Diagram not to scale)

For Earth to begin transfer in Mars' orbit, the

Earth will have an initial velocity change (Δv_a) calculated using the Vis-viva equation:

$$\Delta v_a = v_{trans_a} - v_i, \tag{2}$$

where $v_{trans_a} = \sqrt{\frac{2\mu}{r_{ip}} - \frac{\mu}{a_{trans}}}$ is Earth's velocity at the transfer orbits periapsis and $v_i = \sqrt{\frac{\mu}{r_{ip}}}$ is the initial orbital velocity of Earth and μ is the standard gravitational parameter [6].

Mars Meet Up

Earth is now following the ETO line shown in Fig. 1 as the dashed line. To then match Mars' orbit, another velocity change must occur at the apoapsis of the ETO. The final change in velocity (Δv_b) is calculated using the Vis-viva equation again:

$$\Delta v_b = v_f - v_{trans_b}, \tag{3}$$

where $v_f = \sqrt{\frac{\mu}{r_{fa}}}$ is the final orbital velocity need for Earth to match its orbit with Mars and $v_{trans_b} = \sqrt{\frac{2\mu}{r_{fa}} - \frac{\mu}{a_{trans}}}$ is Earth's velocity at the transfer orbits apoapsis [6].

What a Journey

In addition, the total change in velocity for the overall manoeuvre (Δv) can be calculated through:

$$\Delta v = |\Delta v_a| + |\Delta v_b|, \tag{4}$$

where Δv_a is the initial velocity change into the transfer orbit and Δv_b is the final velocity change for Earth to match Mars' orbit as stated previously.

With these velocity values and as we are assuming the velocity changes occur in one second (Δt), the forces (F) required to inhibit the velocity changes on the mass (m) of the Earth can be calculated [7]:

$$F = \frac{m\Delta v}{\Delta t}, \tag{5}$$

Results

Using Eq. (1), a_{trans} is found to be 198.161×10^6 km. For the Earth to be on track for a Hohmann transfer to Mars' orbit, the initial change in velocity is found to be $\Delta v_a = +3.649$ kms^{-1} using Eq. (2). For Earth to match the orbit of Mars', the change in velocity required is $\Delta v_b = +3.194$ kms^{-1}. Eq. (4) gives a value of $\Delta v = 6.843$ kms^{-1}. The force to cause the initial velocity change is $F_{trans_a} = 2.179 \times 10^{28}$ N and the force to cause the final velocity change is $F_{trans_b} = 1.908 \times 10^{28}$ N.

Conclusion

It is found that the Earth must undergo two velocity increases of magnitude 1 kms^{-1} in perfect alignment with the tangents of both the orbits initial periapsis and final apoapsis to be able to match the orbit path of Mars via Hohmann transfer. Assuming the Earth's velocity changes occur in this way, the forces required are in the magnitude of 10^{28} N. These velocity changes and thus forces applied to Earth could be caused by a collision with an interstellar object, such as a comet or asteroid, travelling with high mass and low velocity; low mass and high velocity or an equal balance of both.

References

[1] W. Hohmann, *The Attainability of Heavenly Bodies* (National Aeronautics and Space Administration, United States, 1960)

[2] J. E. Prussing, Journal of Guidance, Control, and Dynamics. **15**, 1037-1038 (1992).

[3] `https://go.nasa.gov/3ExDtSO` [Accessed 3 October 2021]

[4] `https://go.nasa.gov/3GIEqtk` [Accessed 3 October 2021]

[5] `https://go.nasa.gov/3BCBJG7` [Accessed 3 October 2021]

[6] N. Bannister, *Orbital Manoeuvres* (University of Leicester, United Kingdom, 2020) p. 6.

[7] `https://bbc.in/3byJYs5` [Accessed 4 October 2021]