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A2_7 Santa Doesn't Eat All the Treats

A. Hennessy, S. Manivannan, G. Page, L. Suller

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

In this paper, we consider whether the traditional treats left out for Santa could generate enough energy to power his yearly race to deliver presents around the world. We calculate the energy requirement for the journey using a simplified kinetic energy model, finding that Santa requires 1.42×10^{27} J of energy. Using Einstein's energy-mass relation we calculated the total energy Santa could collect on his trip as 1.72×10^{26} J. This overall leaves a net energy of $\sim 1.25 \times 10^{27}$ J unaccounted for.

Introduction

On the evening of every 24th December, Santa will spend the night delivering presents to the children of the world. Many households leave out a treat for Santa, of cookies, a glass of milk and carrots for the reindeer. Since Santa is obviously unable to eat all these treats himself, we want to calculate if it's possible to convert the matter of these treats into pure energy, using Einstein's mass-energy relation. From this, we are able to compare this energy with the energy required for Santa on his global trip, and see if the provided treats contain enough energy to fully power Santa's yearly trip.

Theory

We first calculated the population of children that require presents. Of the 7.79 billion people on Earth, roughly 1.99 billion ($\sim 25\%$) are under the age of 15 [1]. We assume there are two and a half children per house spread over $\sim$15% of Earth's land surface [2, 3]. The number density n of children Santa must deliver presents to is the child population N divided by the surface area A. We can estimate the average distance between each child d that Santa must travel as, $d = \sqrt{1/n}$.

To calculate the energy Santa requires to visit each child, we first worked out the velocity v,

$$v = \frac{d \times N}{t} \, , \tag{1}$$

as the distance he must travel $d \times N$ in time t (31 hours due to timezones). We modified the kinetic energy equation to sum the kinetic energy between each point of Santa's journey. Subsequent trips take less energy as presents are delivered which is reflected in our calculation of total energy E_1:

$$E_1 = \frac{1}{2} v^2 \left(m_\mathrm{C} + \sum_{x=1}^{N} x m_\mathrm{p} \right) , \tag{2}$$

where m_C is the mass that remains constant throughout (the mass of Santa, reindeers and sleigh), x is the number of presents left on the sleigh and m_p is the average mass of a present.

We can also calculate the energy acquired from food during the trip, using Einstein's energy-mass relation, we formed equation 3 to calculate

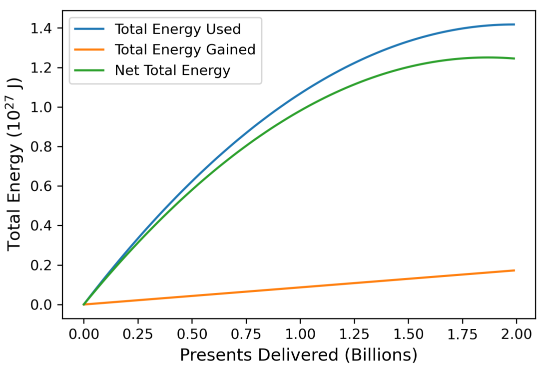

Figure 1: Total energy requirement as a function of the presents delivered throughout the trip. *Blue line*: energy required E_1, *orange line*: energy gained E_2 and *green line*: net energy.

the amount of energy, E_2, Santa can obtain as he makes his trip:

$$E_2 = (m_t N)c^2 \ , \tag{3}$$

where m_t is the mass of treats each house gives Santa. We assume that all of the matter is converted 100% efficiency into kinetic energy. Therefore, E_2 directly gives us the total mass gained by Santa throughout his trip.

From these two values E_1 and E_2, we can simply calculate the total net energy by taking the difference of the two values.

Results

With the population of children in the world and the occupied land area on Earth (22.47 million km^2 [3]), we obtained a 1063 m spacing between each house. The required v to satisfy the distance requirement in 31 hours is 1.89×10^7 m/s, or just over 0.06c. We assume that Santa doesn't require any additional time at each stop to deliver presents.

With the help of his elves, deliveries of presents can be done in batches of 10,000 in an attempt to account for increased densities of population in towns and cities and to reduce the required processing power for the calculation. With m_p as 1 kg and m_C as 1.6 tonnes, the total energy, E_1, is 1.418×10^{27} J.

If each house gives Santa two 20 g cookies, a 25 g glass of milk and nine 0.1 kg carrots (one for each reindeer), then using equation 3 we obtain a value of 1.723×10^{26} J.

The difference of E_1 and E_2 gives a net requirement of 1.245×10^{27} J of energy. How these energy values change as more presents are delivered is shown in figure 1.

Conclusion

We can see that the required energy for Santa's journey is extraordinarily high, on the order of 10^{27} J, whilst the average human energy usage in a year is only on the order of 10^{20} J [4]. Even the conversion of matter into pure energy, a far from realistic method of propulsion, struggles to put a dent into the amount of energy required.

Our calculations are very idealised. We do not account for relativistic effects and assume that acceleration is instantaneous. Our assumptions on the population neglect beliefs and traditions - we assumed that all children celebrate Christmas on the same day. Densities of children will vary on local scales but we approximate this using averages.

Whilst Santa can benefit from utilising all these treats, generating more energy that current human output, there is still a large amount of energy unaccounted for, that can only be powered by Christmas magic.

References

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