Journal of Physics Special Topics

An undergraduate physics journal

A5_6 Bang and the Plug is Gone!: A 3D Analysis

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December 21, 2020

Abstract

In this paper, we perform a 3 dimensional analysis of the trajectory of the plug launched during the Pascal-B nuclear test in 1957. The plug was found to be travelling at 55.55 kms^{-1}, and therefore left the solar system 3 years before the launch of Voyager 1.

Introduction

In a previous paper [1], we began the analysis of the trajectory of the welded steel plug from the Pascal-B nuclear test. We determined the direction in which the plug was launched is critically important to its final trajectory. We also decided one of the assumptions made in that paper, the trajectory is limited to the ecliptic plane, limited the accuracy of our analysis. In this paper, we aim to analyse the trajectory of the plug within a 3D model of the solar system, with the aim of more accurately determining its fate.

Method

In order to perform this analysis, we used the assumptions that the launch velocity was 5 times the escape velocity of the Earth, $5v_E$, and that the Earth's atmosphere provides no resistance. We also used the patched conic approximation of interplanetary trajectories [2]. In a 3D system, latitude and longitude of the launch site, Φ_l and θ_l respectively, and latitude and longitude of the Earth-Sun line at the launch time, Φ_S and θ_S, are needed. Using these the latitude and longitude of the Earth's orbital velocity vector at launch time, Φ_{orb} and θ_{orb}, can then be determined. θ_{orb} is given by $\theta_S + 90°$, as the orbital velocity of Earth is always orthogonal to the Earth-Sun line on the ecliptic plane. Φ_{orb} is given by the point where the ecliptic plane crosses the line of latitude at θ_{orb} at the time of launch. These, along with the launch velocity and the orbital velocity of the Earth, v_l and v_{orb}, were then used to create the velocity vectors, $\boldsymbol{v_l}$ and $\boldsymbol{v_{orb}}$.

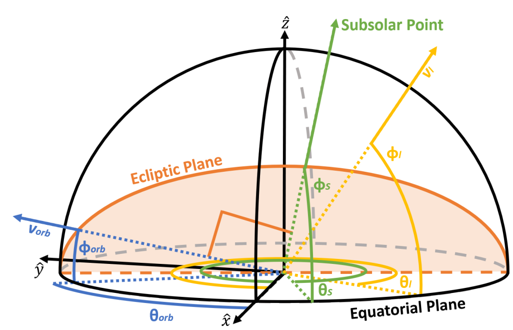

Figure 1: A diagram showing the system on the northern hemisphere, where blue components are for $\boldsymbol{v_{orb}}$, orange components are in the ecliptic plane, green components are for the subsolar point, yellow components are for $\boldsymbol{v_l}$ and black components are the Earth outline.

The velocity of the object with respect to the Sun at the edge of the Earth's gravitational Sphere of Influence (SOI), $v_{SOI,S}$ determines whether the plug would have an interstellar trajectory, when compared with the solar system escape velocity, v_S. $v_{SOI,S}$ is found by resolving $\boldsymbol{v_l}$ and $\boldsymbol{v_{orb}}$ into a single vector. We did this by transforming each vector into a cartesian form to create a coordinate system centred on the the Earth, as seen in Figure 1. This allowed us to convert the velocity vectors by calculating their cartesian components using Equations 1, 2 and 3, which act on a general vector, $\boldsymbol{v}$ with components v, θ and ϕ, and $\phi = 90 - \Phi$, the colatitude.

$$v_x = v\cos\theta\sin\phi \tag{1}$$

$$v_y = v\sin\theta\sin\phi \tag{2}$$

$$v_z = v\cos\phi \tag{3}$$

Once these components had been calculated for both $\boldsymbol{v_l}$ and $\boldsymbol{v_{orb}}$, they were combined to give $v_{SOI,S}$ for the launch using Equation 4,

$$v_{SOI,S} = \sqrt{v_{SOI,x}^2 + v_{SOI,y}^2 + v_{SOI,z}^2} \tag{4}$$

where $v_{SOI,x} = v_{l,x} + v_{orb,x}$, $v_{SOI,y} = v_{l,y} + v_{orb,y}$ and $v_{SOI,z} = v_{l,z} + v_{orb,z}$. We then compared $v_{SOI,S}$ to the escape velocity of the solar system, v_S, as calculated in our previous paper [1]. Finally, to analyse the trajectory, we input the cartesian components of v_l into a GMAT simulation of the solar system at the time of the nuclear test, and ran time forward [3].

Results

For determining Φ_{orb} and θ_{orb}, we took the values of $\Phi_S = 158.4°$W and $\theta_S = 9.916°$N [5]. This gave us values of $\Phi_{orb} = 248.4°$W and $\theta_{orb} = 21.36°0$N. These were combined with $v_{orb} = 29.78$ kms^{-1}[1], generating $\boldsymbol{v_{orb}}$. Values of $\Phi_l = 116.2°$W and $\theta_l = 37.12°$N [4] were found and combined with $v_l = 55.93$ kms^{-1} to generate $\boldsymbol{v_{orb}}$. These vectors then gave the cartesian components: $v_{l,x} = -19.68$ kms^{-1}, $v_{l,y} = 40.02$ kms^{-1}, $v_{l,z} = 33.75$ kms^{-1}, $v_{orb,x} = -29.89$ kms^{-1}, $v_{orb,y} = 14.24$ kms^{-1} and $v_{orb,z} = 10.85$ kms^{-1}. Running these values through Equation 4, gave us a value of $v_{SOI,S} = 55.55$ kms^{-1}. When the components of $\boldsymbol{v_l}$ were input into GMAT, with a start time of 22:35 on 27th August 1957 [6], Figure 2 was generated. The simulation determined the plug escaped the solar system on 18th July 1974.

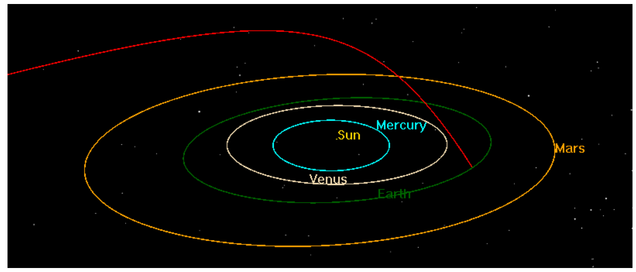

Figure 2: The GMAT visualisation showing the red hyperbolic orbit of the plug as it escapes the solar system.

Conclusion

In conclusion, the value of $v_{SOI,S} = 55.55$ kms^{-1} is approximately 13.44 kms^{-1} larger than the value of $v_S = 42.11$ kms^{-1} [1]. Therefore our calculations show that the plug escaped the solar system, which is backed up by our GMAT simulation. In addition, the simulation showed that the plug left the solar system over three years before the next object destined to leave the solar system, Voyager 1, was even launched.

References

[1] J. MacQuillin, J.F.B.Q. Kerr, I. Priest and J. Musk, *A5_5 Bang and the Plug is Gone!*, PST 19, (2020)

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[4] `https://tinyurl.com/y6y33fq2` [Accessed 1 December 2020]

[5] `https://tinyurl.com/yxee9gs7` [Accessed 1 December 2020]

[6] `https://tinyurl.com/y3katyts` [Accessed 1 December 2020]