Journal of Physics Special Topics

An undergraduate physics journal

A5_5 Bang and the Plug is Gone!

J. MacQuillin, J.F.B.Q. Kerr, I. Priest, J. Musk

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 21, 2020

Abstract

In this paper, we analyse whether a steel plug launched vertically upward from an underground nuclear test could have escaped the solar system. We found that while the plug has the potential to reach interstellar space, the direction in which the plug was launched plays a significant role in it's fate, and therefore we propose writing a follow up paper to determine the plug's final fate.

Introduction

Pascal-B was a 1957 investigation into methods for containing the fallout of nuclear testing and was the second ever underground nuclear test. The test involved detonating a small nuclear device within a vertical borehole which was sealed at the surface with a welded steel, manhole cover-like, plug. After the disappearance of this plug during a previous test, a high speed camera was used to attempt to capture the plug's escape in order to understand it's fate. Upon review of the footage, it was found to have been launched upward at an astonishing rate. This paper aims to determine the effects of this launch speed, and determine the validity behind the claims that it may have been the first object to escape the solar system.

Method

Robert Brownlee, the physicist leading the project, stated in an interview that the plug was launched at 5 times the escape velocity of Earth [1], $5v_E$. This provided us with our first assumption. We also assumed that the Earth's atmosphere provided no drag, and that the trajectory was limited to the ecliptic plane.

To go about determining the effects of the launch speed, we used the patched conic approximation for interplanetary trajectories [2]. The first step of our method was to determine the velocity of the plug at the edge of the Earth's Sphere of Influence (SOI) with respect to the Earth, $v_{SOI,E}$. We did this by rearranging the equation for determining the launch velocity of an object leaving the Earth, v_l. This gave us Equation 1,

$$v_{SOI,E} = \sqrt{v_l^2 + \frac{2\mu_E}{R_E}} \tag{1}$$

where R_E is the radius of the Earth and $\mu_E = GM_E$, where G is the universal gravitational constant and M_E is the mass of the Earth. Once the plug has reached the edge of the Earth's SOI, the Earth is no longer the dominant gravitational body. This means that the velocity of the plug at the edge of Earth's SOI with respect to the Sun, $v_{SOI,S}$, needs to be determined. This was done by considering the direction in which the plug leaves Earth's SOI, as the plug will also have the velocity of the Earth's orbit, v_{orb}, in the reference frame of the Sun. To simplify this, we considered a best and worst case scenario for escaping the solar system. This is shown in Figure 1 as scenario A and B respectively.

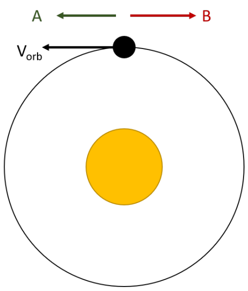

Figure 1: A diagram centred on the Sun, showing the direction of the Earth's Orbital Velocity and the direction of $v_{SOI,E}$ in scenario A and B. *NOT to scale.*

Scenario A shows the direction of $v_{SOI,E}$ as being parallel to that of v_{orb}. This would be the best case scenario for the plug to leave the solar system, as all the velocity is in the same direction, resulting in the largest possible combined magnitude. In this case, $v_{SOI,S}$ is the maximum possible value, v_A, and is given by Equation 2.

$$v_A = v_{SOI,E} + v_{orb} \tag{2}$$

Scenario B shows the exact opposite case to Scenario A, which would result in the minimum possible $v_{SOI,S}$, v_B, and is given by Equation 3.

$$v_B = v_{SOI,E} - v_{orb} \tag{3}$$

Once v_{SOI} values have been calculated, they must be compared to the escape velocity of the solar system at the Earth's orbital distance, v_S. This is given by Equation 4,

$$v_S = \sqrt{\frac{2GM_S}{a_E}} \tag{4}$$

where M_S is the mass of the Sun and a_E is the semi-major axis of the Earth's orbit.

Results

For the calculation of $v_{SOI,E}$, we took $v_l = 5v_E$, where $v_E = 11.19$ kms^{-1} [3]. We then used the values $v_l = 55.93$ kms^{-1}, $\mu_E = 3.986 \times 10^5$ km^{3}s^{-2} and $R_E = 6371$ km [3]. This produced a value of $v_{SOI,E} = 54.80$ kms^{-1}. From here we calculated values of v_A and v_B with Equations 2 and 3 respectively, both of which required $v_{orb} = 29.78$ kms^{-1} [3]. This gave us values $v_A = 84.58$ kms^{-1} and $v_B = 25.02$ kms^{-1}. Finally we needed to calculate v_S, which needed values of $M_S = 1.989 \times 10^{30}$ kg [4] and $a_E = 149.6 \times 10^6$ km [3]. These gave the value of $v_S = 42.11$ kms^{-1}.

Conclusion

These results mean that in Scenario A, the plug would comfortably escape the solar system, whereas in Scenario B, the plug would enter a retrograde orbit around the Sun, passing between Mercury and Venus at perihelion. Therefore it can be seen that the direction that the plug is launched at is crucial to it escaping the solar system. Our assumption, that the atmosphere provided no drag, is a reasonable assumption, since the plug only spends 1.789 s in the atmosphere travelling at it's launch velocity. In that time, the atmosphere would have very little chance to affect the velocity of the plug. Our final assumption, that the trajectory is limited to the ecliptic plane, is very poor. This is because the launch site in Nevada falls North of the Tropic of Cancer, the highest line of latitude that sits on the ecliptic. In conclusion, it is a perfectly valid claim that the plug may have escaped the solar system. However, a follow up paper will allow us to more definitively determine the final fate of the plug using a 3D model of the solar system to account for the latitude of the Nevada launch site.

References

[1] `https://tinyurl.com/BrownleeBI` [Accessed 29th November 2020]

[2] R.R. Bate, D.D. Mueller, J.E. White, *Fundamentals of Astrodynamics* (New York, 1971), Ch. 8.

[3] `https://tinyurl.com/yc2fdlos` [Accessed 29th November 2020]

[4] `https://tinyurl.com/y3bfu7nv` [Accessed 29th November 2020]