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A1_11 Mars' Magical Cows

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Abstract

The Kármán line defines the edge of the atmosphere and in *A1_5* we found this would be 210 km above the Martian surface for a methane atmosphere. Accordingly, modelling a plane parallel atmosphere we deduce that reproducing cows could produce the total mass of methane required for this atmosphere within 56.8 yrs. The total number of cows this would require is significantly larger than the surface area of Mars and so we also determined that for the maximum number of cows able to be on the Martian surface, they could create this methane atmosphere in ~ 1200 yrs.

Introduction

Considering the astonishing rate of methane production by Earth cows, we look to determine the feasibility of using magical cows - that have the ability to live on Mars - to produce a methane atmosphere on Mars. We begin by calculating the total mass of methane contained within the Martian atmosphere, which is bounded by the Kármán line, calculated in our previous paper *A1_5* to be at 210 km [1]. Then we determine the time it would take for methane levels to fill this volume by considering the reproduction rate of our Martian cows, alongside the amount of methane produced by each individual cow.

Theory

In the previous paper, *A1_5*, the Law of Atmospheres (their Eq. 5) and the ideal gas law were used to define the particle number density n as a function of height,

$$n = \frac{P_0}{kT} e^{(-(\rho_0/P_0) g_M y)} \tag{1}$$

where ρ_0 and P_0 are the density and pressure at the surface respectively, k is the Boltzmann constant, T is the temperature and g_M is the acceleration due to gravity on Mars (values and further definitions of these constants can be found in [1]). If we integrate Equation 1 up to the distance of the Kármán line, then the number density can be expressed as a surface density σ,

$$\sigma = \int_0^{210\,\mathrm{km}} \frac{P_0}{kT} e^{-(\rho_0/P_0) g_M y}\,\mathrm{d}y \tag{2}$$

which is approximately $1 \times 10^{30}\,\mathrm{m}^{-2}$. Assuming a plane parallel atmosphere we use the volume of the atmospheric shell V_A ($\sim 3.2 \times 10^{19}\,\mathrm{m}^3$) and its respective height y (the 210 km Kármán line) to calculate the resulting surface area S_A,

$$S_A = \frac{V_A}{y} \sim 1.5 \times 10^{14}\,\mathrm{m}^2. \tag{3}$$

Therefore the total mass of methane contained within the atmosphere M_M is expressed as

$$M_M = S_A m_p \sigma \sim 4 \times 10^{18}\,\mathrm{kg} \tag{4}$$

where the value of m_p is 2.664×10^{-26} kg [1].

Next, we consider how quickly Martian cows could produce this methane mass assuming we start with 2 reproducing cows (N_0) flown to Mars from Earth. The reproduction rate is given as

$$N = N_0 e^{\lambda t} \tag{5}$$

where N is the total number of cows (at any respective time), and t is the time required to create the atmosphere which we are looking to determine. From a reproductive success rate of 92% per year we find the value of λ to be ~ 0.65 [2]. Equation 5 can be substituted into the rate equation,

$$R_T = N R_M \tag{6}$$

where R_T is the total rate of the production of the atmosphere and R_M is the rate at which a single cow produces methane assumed to be constant for all ages of cow at a value of $\sim 120\,\mathrm{kg\,yr^{-1}}$ [3]. The rate of atmosphere production can also be expressed as $R_T = \mathrm{d}M/\,\mathrm{d}t$ such that if we rearrange Equation 6 and integrate

$$\int_0^{M_M} \mathrm{d}M = R_M N_0 \int_0^{t_f} e^{\lambda t}\ \mathrm{d}t \tag{7}$$

then we can find an expression for the time taken for the cows to produce the atmosphere,

$$t_f = \frac{1}{\lambda} \ln\left(1 + \frac{\lambda M_M}{R_M N_0}\right). \tag{8}$$

Results

From our previous definition of the Kármán line and the reproductive rate equation, we were able to determine that it would take 56.8 yrs for the cows to reproduce and produce the total mass of methane required for the atmosphere. We then calculated the total number of cows that would be alive by the end of this time period by substituting the time back into Equation 5, which gave 2.2×10^{16} cows. This number of cows is two orders of magnitude higher than the surface area of Mars which is $\sim 1.4 \times 10^{14}\,\mathrm{m}^2$ meaning not all of these cows could fit on the surface at one time. To fit all of these cows, they would have to be stacked in stacks of a 100 cows each. With an individual cow height of 1.8 m, the total height of these stacks is 180 m, effectively increasing the radius of Mars by this amount [4]. However, this is also small in comparison to the atmosphere height and so would not significantly affect it.

On the other hand, if we assign an area of $\sim 5\,\mathrm{m}^2$ to each cow then the maximum number of cows on the surface at one time is capped at $\sim 2.8 \times 10^{13}$. In this case we find it would take ~ 1200 yrs for enough methane to be produced to form the atmosphere, assuming no gas is lost in the process.

Conclusion

By considering the amount of methane contained within an atmosphere where the edge is defined by our previously calculated Kármán line, we first determined that a total of $\sim 2.2 \times 10^{16}$ cows would take 56.8 yrs to create the atmosphere. As this value was much larger than the available space on the Martian surface we then considered how long it would take for the maximum number of cows to produce the required mass of methane and this was found to be ~ 1200 yrs. Overall our model is significantly simplified as there are other aspects that would need to be taken into consideration such as the life span of the cow, the survival rate of a calf and the cows' methane production rate with respect to age. In the future, if we were to settle on Mars, magical cows are a potentially viable method of atmosphere production.

References

[1] Y. Davis, N. Drewes, T. Lomax, S. Norman *A1_5 Methane Mars*, PST 19, (2020).

[2] `https://extension.psu.edu/reproduction-by-the-numbers` [Accessed 2 December 2020].

[3] `https://phys.org/news/2019-07-potential-methane-cows.html` [Accessed 2 December 2020].

[4] `https://www.dimensions.com/element/dairy-cows` [Accessed 3 December 2020].