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A1_9 The Power of Christmas Cheer

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Abstract

Christmas cheer is a powerful thing: in the film *Elf* it is shown to be powerful enough to make Santa's sleigh fly. In this paper we calculate the energy generated by a single person's worth of Christmas cheer, by considering the velocity required to lift the sleigh off the ground. This value is calculated to be 1.6×10^4 J. We then use this value to discuss the viability of Christmas cheer as a form of renewable energy.

Introduction

A key part of Christmas in households across the globe is Will Ferrell's portrayal of Buddy the Elf in the movie *Elf* [1]. Throughout the film Buddy teaches us some very important lessons about Christmas and family. However, the most important lesson we learn, and the one that ultimately saves the day, is rule number three of elf school: the best way to spread Christmas cheer is singing loud for all to hear. It is this Christmas cheer that powers Santa's sleigh at the end of the film when his engine gets knocked off by a collision with a statue. Using the fact that the sleigh is only powered by the Christmas cheer, we will calculate the energy of a single person's Christmas cheer.

Theory

In order to quantify the energy of Christmas cheer from one person, we need to model Santa's sleigh as an aircraft. This allows us to use the following equation for the force of lift required for it to fly,

$$L = \frac{1}{2} C_l A \rho v^2 \tag{1}$$

where C_l is the lift coefficient, ρ is the density of air, v is the velocity and A is the area of the wings. This is then equated to the force of gravity, $F = mg$, as that is the minimum required to raise the sleigh from the ground into the air. We then solve for velocity which gives

$$v = \sqrt{\frac{2mg}{A\rho C_l}}. \tag{2}$$

This velocity is then used to calculate the kinetic energy of the sleigh, from which we can calculate the energy each person contributed toward the total energy by dividing by the total number of people present at the time of singing.

Since the Christmas cheer is spread through song, a source of this energy might be the energy contained within these sound waves. For a volume of air of $1\,\mathrm{m}^3$, the energy of the sound wave is $E_S = 0.5\rho\omega^2 s_0^2$, where ω is the frequency and s_0 is the displacement of the air.

Results

To model the sleigh as an aircraft a Saab 340 was chosen, as it is a relatively small plane. The plane has a wingspan of $41.8\,\mathrm{m}^2$ and as it is a twin engine propellant plane it has a lift coefficient, C_l, of roughly 1.6 [2, 3]. Furthermore, the mass of the sleigh was taken as 816 kg (assuming the wings added no additional mass) and the mass of a caribou (reindeer) is taken as 320 kg [4, 5]. Therefore, the total mass of the sleigh and reindeer combined is 3696 kg. Using these values and Eq. 2 we find the take-off velocity to be $29.8\,\mathrm{m\,s^{-1}}$ and the total kinetic energy of the sleigh to be 1.6×10^6 J.

While the singing of Christmas carols was broadcast on the news in the film and many audience members sing along it is unknown just how many people joined in. Therefore, we assume that only the people at the scene of the singing affect the flight of the sleigh. The number of people at the scene is roughly 100. Therefore, one persons worth of Christmas cheer is equal to 1.6×10^4 J.

Finally, for the frequency range of the human voice of 125−8000 Hz and $s_0 \sim 1$ mm, the energy produced by a single person singing is roughly 1600 J [6]. As this is a tenth of the previous value, it is obvious that almost all of the energy is supplied by the Christmas cheer.

Discussion

To reach every child in one evening Santa would need to use of time dilation. In order for this to occur he would need to be travelling at speeds close to the speed of light. We calculate a value of energy required to take advantage of time dilation by using $E = (\gamma - 1)mc^2$, where γ is the Lorentz factor. If we assume Santa is travelling at 99% of the speed of light he would require 2.02×10^{21} J and 1.26×10^{17} people for Santa to fly fast enough.

Another use of Christmas cheer could be as a renewable energy source for powering homes. The average house in the UK consumes 12 000 kW h which is 4.32×10^{10} J over the course of a year [7]. Therefore, it would require 2.7 million people to power a single average household. These two comparisons show that Christmas cheer alone is not a great source of energy. However, it is still able to replace some amount of energy from more harmful sources such as fossil fuels.

We have also made some assumptions in order to calculate the value of energy. Firstly, the airflow and the mass between a plane and Santa's sleigh are very different and thus the lift generated by Santa's sleigh would be less compared to the plane at the same speed due to its non-optimized shape. Furthermore, air resistance has not been taken into account, in which case the energy required to reach the take-off velocity would be higher.

Conclusion

One person's worth of Christmas cheer, 1.6×10^4 J, may not be the new revolutionary form of energy generation that will prevent global warming and is not able to power Santa's relativistic needs. However, it shows its true power in other ways. Christmas cheer brings out the best in people and reminds that Christmas is all about spending time with loved ones just like Buddy finally gets to do at the end of the film, and while we may not all believe in Santa, we should all believe in the power of Christmas cheer.

References

[1] *Elf*, New Line Cinema (2003).

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