Journal of Physics Special Topics

An undergraduate physics journal

A1_8 O Christmas Tree, How Warm Are You?

Y. Davis, N. Drewes, T. Lomax, S. Norman

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 17, 2020

Abstract

In this paper we examine the warmth of a Christmas tree through the thermal energy radiated by a string of electrical light bulbs and candles. We model the candles as a black body to infer their temperature as 700 K and their total radiative power (for 25 candles) as 1700 W. On the other hand, due to the heat lost in the filament of the incandescent bulbs, 150 of these radiate at 15 000 W. The equivalent black body temperature of the tree is 110 °C for the electric lights and −50 °C for the candles. Due to radiating more heat, the electrically lit tree will be drier and warmer, however since the candles are open flames, they are more likely to incinerate the tree.

Introduction

While the most widespread way to light the Christmas tree is using electrical fairy lights, in our house we have traditionally used real candles. This creates a far more cozy atmosphere and saves on the electricity bill. However, there is also an increased fire risk, especially through cat accidents ('catcidents') – one author's father's cat once jumped into their tree, toppling it. Nevertheless, the incandescent light bulbs with the glowing filaments also create large amounts of heat. Therefore, we investigate whether the traditional candles actually radiate more power than incandescent light bulbs and which one therefore warms the tree more. We will model the candles as a black body to infer their temperature and their luminosity, and the light bulbs as a simple resistor in a circuit.

Method

We want to know the thermal energy radiated by both the candles and the incandescent light bulbs. Therefore, we model the candles as a black body to find their equivalent black body temperature. The Sun, which has its peak in the visible spectrum, is at a temperature of 5800 K. Since the candle will be at a much lower temperature T, we are looking at the high frequency tail of the black body spectrum $B_\nu(\nu, T)$ (ν is frequency), and aiming for 10% of the radiated flux to be in the visible spectrum ($\nu_1 \rightarrow \nu_2$):

$$0.1 = \frac{\int_{\nu_1}^{\nu_2} B_\nu(\nu, T)\,\mathrm{d}\nu}{\int_0^\infty B_\nu(\nu, T)\,\mathrm{d}\nu}. \quad (1)$$

As we are operating in the high frequency tail of the spectrum, such that $h\nu \gg kT$, we can apply Wien's Law $B_\nu(\nu, T) = 2h\nu^3/c \cdot \exp(-h\nu/kt)$. In addition, the denominator in Eq. 1 is simply given by σT^4. Therefore, integrating the numerator in Eq. 1 and assuming all terms with ν^2 and smaller orders are negligible, gives

$$0.1\sigma T^3 = \frac{2k}{c}\left(\nu_1^3 \mathrm{e}^{-\nu_1 h/kT} - \nu_2^3 \mathrm{e}^{-\nu_2 h/kT}\right). \quad (2)$$

The visible spectrum is roughly defined as the wavelengths between 380 nm and 750 nm, which corresponds to frequencies of $\nu_1 = 4 \times 10^{14}$ Hz and $\nu_2 = 8 \times 10^{14}$ Hz. Using these values, the numerical solution of Eq. 2 gives the black body temperature of the candle as $T \sim 700$ K. The power radiated by this candle P_C is its black body luminosity:

$$P_C = L = 4\pi r^2 \sigma T^4 \qquad (3)$$

where r is the radius of the black body.

Furthermore, to evaluate the thermal power radiated by the incandescent light bulbs we use the resistance R of the filament inside the bulb. As a current flows through this wire, it increases in temperature and starts glowing. The thermal power P_B lost in this resistor is

$$P_B = \frac{V^2}{R}, \qquad (4)$$

where V is the voltage supplied to the bulb.

Results

Here we compare the power radiated by the candles and the bulbs. We model the candles as a black body with an equivalent temperature of 700 K and a radius of 2 cm. According to Eq. 3, this gives a power per candle of 68 W. A typical, room-sized Christmas tree can hold between 20-25 candles which in total radiate at 1360-1700 W. On the other hand, a light bulb with typical resistance 144 Ω at the US household voltage of 120 V will have a thermal power loss of 100 W, according to Eq. 4 [1]. For fairy light strings of about 150 bulbs, this means that the whole tree is radiating at 15 000 W [2]. Clearly, the electric lights lose (and use) far more energy than the candles.

Further, we can also now find the equivalent black body temperature of the tree and the lights, by solving Eq. 3 for the temperature. A room-sized Christmas tree that is about 2 m high has a radius of 1 m. The tree with the candles and a maximum luminosity output of 1700 W will have a black body temperature of 220 K (−50 °C), while the tree with the lights will have a temperature of 380 K (110 °C).

Conclusion

As incandescent light bulbs are very inefficient and lose a lot of energy in heat, they tend to warm up the Christmas tree more than candles, as shown by the equivalent black body temperatures: 110 °C versus −50 °C. This is not the actual temperature but a measure of the radiative power. Nevertheless, as the electrically lit tree radiates more heat it will be warmer and dry out faster, and therefore catch on fire more easily in the presence of open flames or electric sparks. Non-withstanding, there is a greater danger from the candles' open flames. One of the reasons for the bulbs' greater power output is that there are typically about 7 times more bulbs than candles on the tree. In addition, we heavily approximated the candles and the tree as ideal black bodies. In reality, candles have a temperature of ~ 1300 K but a comparable power of ~ 80 W since they do not radiate as efficiently as black bodies [3, 4]. Nevertheless, the large difference between the powers and temperatures for both trees shows that the electric lights clearly warm the tree more. Barring the occasional 'catcident', the electrically lit Christmas tree is more likely to catch on fire than the candle lit one, due to its higher thermal radiation and temperature.

References

[1] `https://www.fluke.com/en-us/learn/blog/energy-efficiency/say-goodbye-to-the-incandescent-lightbulb` [Accessed 1 December 2020].

[2] `https://www.realsimple.com/holidays-entertaining/holidays/christmas/how-many-feet-christmas-lights-tree` [Accessed 2 December 2020].

[3] `https://hypertextbook.com/facts/1999/JaneFishler.shtml` [Accessed 16 December 2020].

[4] `https://www.thenakedscientists.com/articles/questions/how-many-watts-burning-candle` [Accessed 16 December 2020].