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P1_6 The Revenge of Pluto

K. Roberts, N. Holt, J. Murphy, J. Tranter

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

Pluto is a bit annoyed at being demoted to a dwarf planet, and it is after revenge. It plans to collide with and destroy the Earth by adjusting its own orbit. In this paper, the minimum value for the velocity of Pluto (relative to the Earth) will be calculated, in order for it to destroy the Earth, as well as the minimum separation between the two bodies at which if the system was released from a stationary state, gravitational forces would cause the acceleration of the bodies, and so, destruction.

Introduction

After being declassified as a planet on the 24th of August, 2006 [1], anthropomorphised Pluto isn't too happy about the situation, and has decided to take its revenge on the residents of Earth by adjusting its orbit (using super secret magic rockets its been hiding from us for this very occasion), to collide into Earth, and such turn our planet into the solar system's newest asteroid belt. To do this, Pluto will need to ensure that its velocity relative to its target is great enough on impact, for the gravitational binding energy of the Earth to be overcame, and thus destroy it. In this paper, the minimum needed impact velocity of Pluto as it impacts the Earth will be found. The distance needed between the two bodies will be found so that the Earth is destroyed, in the case that they are released from rest (relative to each-other). This will be done by finding the gravitational potential energy needed to be present between the Earth and Pluto.

Because of the scales and possible velocities involved, general relativistic effects will be considered in the following calculations.

Method and Results

To find the minimum velocities and energies needed, the magnitude of energy needed to destroy the Earth needs to be needs to be known, which will be taken to be the gravitational binding energy (the energy needed so that the constituent parts of the Earth are no longer gravitationally bound to each-other) as gravitational forces are assumed to be dominant on the Earth as opposed to inter-molecular. For this paper, all planetary bodies will be taken to be perfect spheres with uniform densities, and having negligible atmospheres. In this case, value of the magnitude of the gravitational binding energy of a body is given by the following equation:

$$E_b = \frac{3}{5}\frac{GM_T^2}{R_s}[2] \quad (1)$$

Where E_b is the binding energy of the system, G is the gravitational constant, M_T is the total mass of the system and R_s is the initial radius of the system. Because the Earth and Pluto are colliding, M_T will be equal to the total mass of Earth and Pluto (assuming they initially merge), and R_s will be the total radius of the merged system (assuming all materials are incompressible and have uniform density). The total mass of Earth and Pluto can be calculated to be equal to $5.99 * 10^{24} kg$ [3], and the total radius of the combined system can be found to be $6.38 * 10^6 m$ [3]. Thus, the binding energy of the whole system can be calculated by the use of equation (1) as $2.25 * 10^{32} J$.

Because this value is now known, it can be equated to the kinetic energy of Pluto at the point of merging with the Earth, and rearranged to find the velocity at impact. The relativistic kinetic energy is given by:

$$KE_r = m_o c^2 \left(\frac{1}{(1 - \frac{v^2}{c^2})^{1/2}} - 1\right)[4] \qquad (2)$$

Where KE_r is the kinetic energy of Pluto m_0 is the rest mass of Pluto, c is the speed of light and v is the velocity of Pluto as measured from Earth. Doing this (while inputting the rest mass value for Pluto ($1.309 * 10^{22} kg$[3])) produces a velocity at impact from the reference frame of Pluto of $57.48 ms^{-1}$.

In order to find the spacial separation of the Earth and Pluto, at which the acceleration due to the gravitational forces between the two would cause an energetic enough impact, the gravitational binding energy of the system (the combination of Earth and Pluto after merging) is once again needed, which was earlier calculated to be $2.25 * 10^{32} J$. Equating this to the equation for gravitational potential energy (assuming point mass sources) between two stationary bodies (as is the initial case in this system), which is:

$$E_{Grav} = \frac{G M_E M_P}{r}[4] \qquad (3)$$

Where G is the gravitational constant, M_E is the rest mass of the Earth ($5.97 * 10^{24} kg$ [3]), M_p is the rest mass of Pluto ($1.31 * 10^{22} kg$[3]) and r is the separation between the two bodies, and then rearranging for r gives an initial separation value of $23275 m$ (which is much less than the current distance from Earth to Pluto).

Conclusion

In summary, in order for Pluto to destroy the Earth, the velocity that it would need to impact the Earth at would be very low ($57.48 ms^{-1}$ (from the reference frame of Pluto)), and also the distance it would need to be "dropped"/"released" from, would also be low ($23275 m$), therefore showing the fragility of the Earth in such an event. This situation does ignore how Pluto would come into contact with the Earth, as well as the intermolecular and thermal heating of the resulting mass which would change the results (as only gravitational forces have been considered). By taking these extra forces into account, this would show the Earth to be much more resilient from Pluto's attacks.

References

[1] `https://www.space.com/why-pluto-is-not-a-planet.html` [Accessed 02/12/2020]

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[3] `https://solarsystem.nasa.gov/planets/dwarf-planets/pluto/by-the-numbers/` [Accessed 02/12/2020]

[4] `http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/releng.html` [Accessed 02/12/2020]