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P2_4 The Original Pyramid Scheme

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Abstract

The pyramids that sit atop the Giza plateau are a fascinating example of ancient megalithic construction. There are numerous theories explaining their construction so this paper attempts to test some of the claims made by egyptologists. It was found that the Great Pyramid of Giza could be built by 20,000 men, over the 20 year time period suggested, with an excess intake of only 17 kcalories per worker per day.

Introduction

The Egyptian pyramids have long perplexed egyptologists as to the methods of their construction. Theories range from criss-crossing slopes to interior ramps, with none fully explaining the remains we see on the Giza plateau today. It is generally accepted that the largest pyramid, the Pyramid of Khufu, was built on a timescale of about 20 years with a workforce of anywhere from 20,000 to 100,000 men [1]. This paper investigates the veracity of these claims by calculating the excess number of calories each worker would have to consume in order to have the energy and strength to raise the 2.3 million limestone bricks that comprise the pyramid.

Theory and Method

In order to test the claims made by archaeologists, we calculate how much energy is required to construct the pyramid. This is done by working out the gravitational potential energy of a layer of blocks and integrating over the height. The equation for the total energy we arrived at is given as

$$E = \rho g \left(\frac{a_0}{h_0}\right)^2 \int_0^{h_0} h(h_0 - h)^2 dh \qquad (1)$$

where ρ is the density of limestone [2], g is the gravitational acceleration, a_0 is the base length, h_0 is the height, and h is the variable height.

For this example, the pyramid is assumed to be a perfect square based pyramid made purely of limestone (despite the real thing actually having eight sides and numerous tunnels and chambers!). Once this calculation is done, we convert the energy into a number of calories and divide over the 20 year time period and the number of workers. The efficiency of the human body is also taken into account as we can only convert about 25% of calorific energy into actual work [3].

Results and Discussion

The plot (Figure 1) of the integrand of (1) shows that the most difficult part of the pyramid would be just below the centre. The base has a fairly low energy requirement, despite the large number of blocks it contains, as the blocks would not need to be raised very high. The top

also has a low energy requirement as the blocks must be raised very high but there are very few of them. Summing over the entire height of the pyramid results in equation (2).

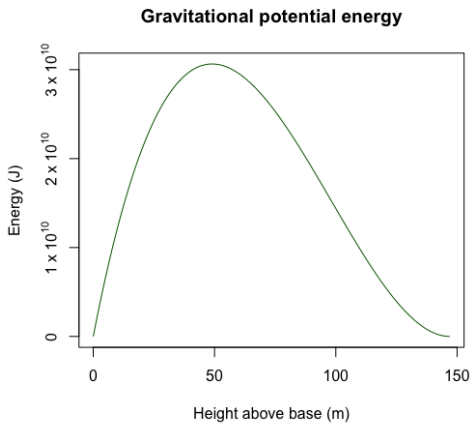

Figure 1: Gravitational potential energy required to raise a horizontal cross section of the pyramid up to the height it sits at. From the base of the pyramid up to the highest point at 147 m

$$E = \frac{\rho g a_0^2 h_0^2}{12} \tag{2}$$

Substituting the values specific to Khufu's Pyramid into (2) gives a total energy of 2.5×10^{12} J. This was then divided over the 20 year construction period and shared between each worker. Converting into kcalories gave the plot seen in Figure 2.

This plot clearly shows that with the large number of workers suggested, the kcalorie excess that each person would have to take in, in order to do work, is minimal and completely reasonable. The lower limit of 20,000 workers results in the need for only 17 kcal extra per worker per day and would only reach the slightly high value of 1000 kcal for a workforce of just 332 people. Even this is plausible so long as the workers could find time to lift the stones around all of their meals!

However, there would be many other factors that increase the energy requirements for the workers that aren't considered in this paper. The

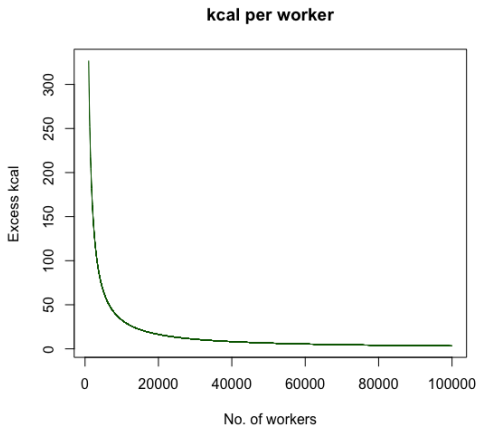

Figure 2: Excess calories required per worker per day.

model we used assumes that the stones are ready and waiting to simply be raised into place. In reality, workers would have to cut and carve each block of limestone and then transport them all to the plateau before construction. Factors such as these could raise the energy requirements by a significant factor and their lack of inclusion may explain why the calorie excess is so low for the size of the workforce suggested by archaeology.

Conclusion

We may never know the exact mechanisms or techniques employed in the construction the pyramids of Giza, but that won't stop us speculating. We conclude that for the 20 year time period suggested, the human energy requirements could easily be met by even the minimum suggested workforce of 20,000 workers by each of them eating an extra 17 kcal per day.

References

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